

The Quadrupole Cusp: A  
Universal Accelerator,  
or  
*From Radiation Belts to  
Cosmic Rays*

Robert Sheldon

NSSTC Seminar, March 19, 2004

# Outline

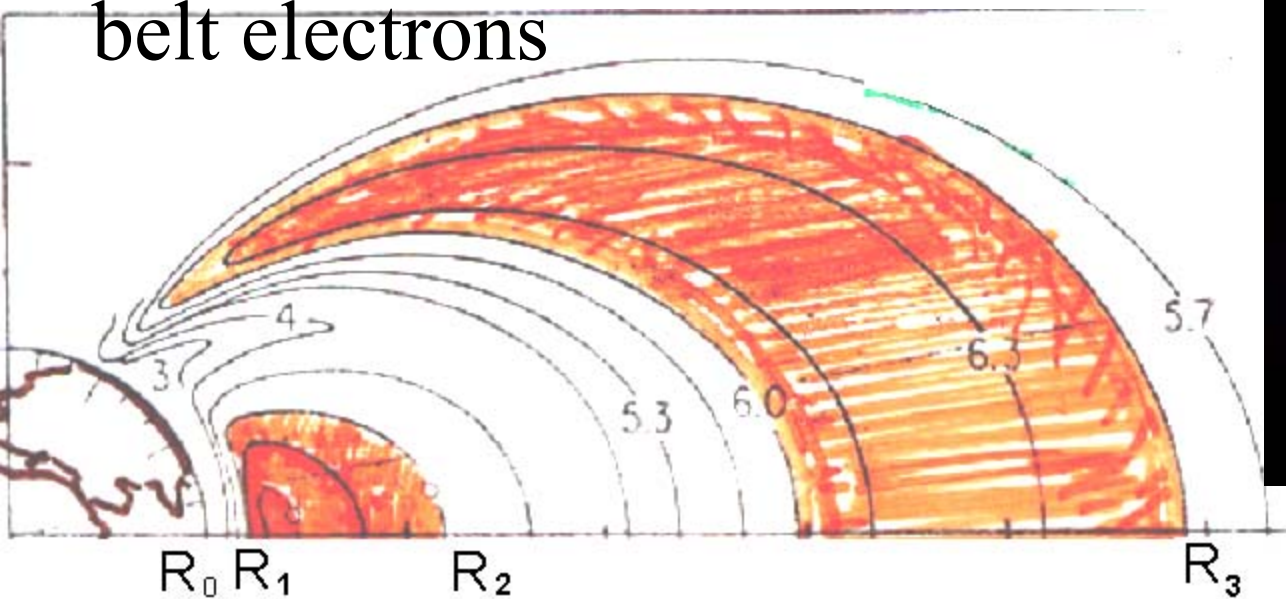
- Quadrupoles
  - Plasma Thermodynamics
  - Acceleration Mechanisms and Entropy
    - shocks, waves, reconnection and dipole
    - Why a trap improves acceleration efficiency
  - How a Quadrupole Traps & Accelerates
  - Signatures of Quadrupole Acceleration
- Application 1: Outer Radiation Belt Electrons
- Application 2: Galactic Cosmic Rays

# Introduction

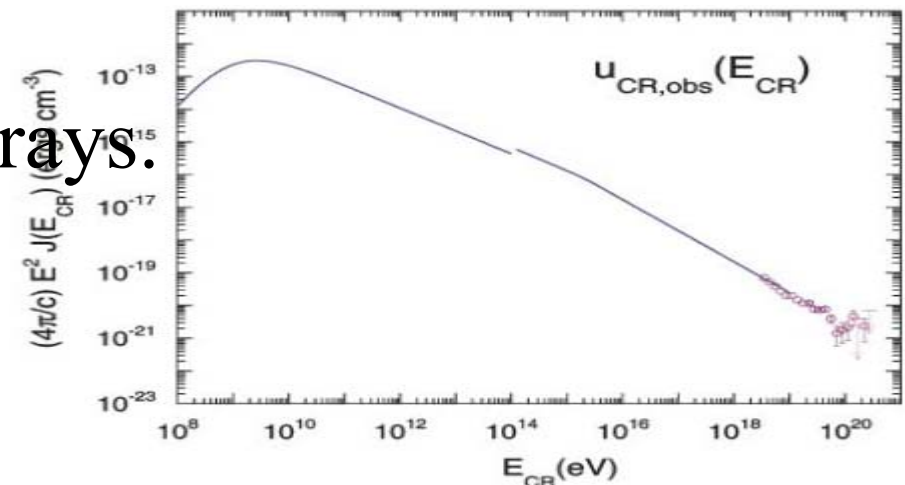
- Why is studying acceleration mechanisms important?
  - Fundamental physics (e.g. Nobel Prizes)
  - Usefulness in medicine, science, technology
    - Radiation therapy, advanced light sources, microwave ovens etc.
  - Prediction of harmful radiation to satellites and humans in space.
    - NASA goals, even Moon missions!
  - Plain fun.

# Two “unsolved” physics problems

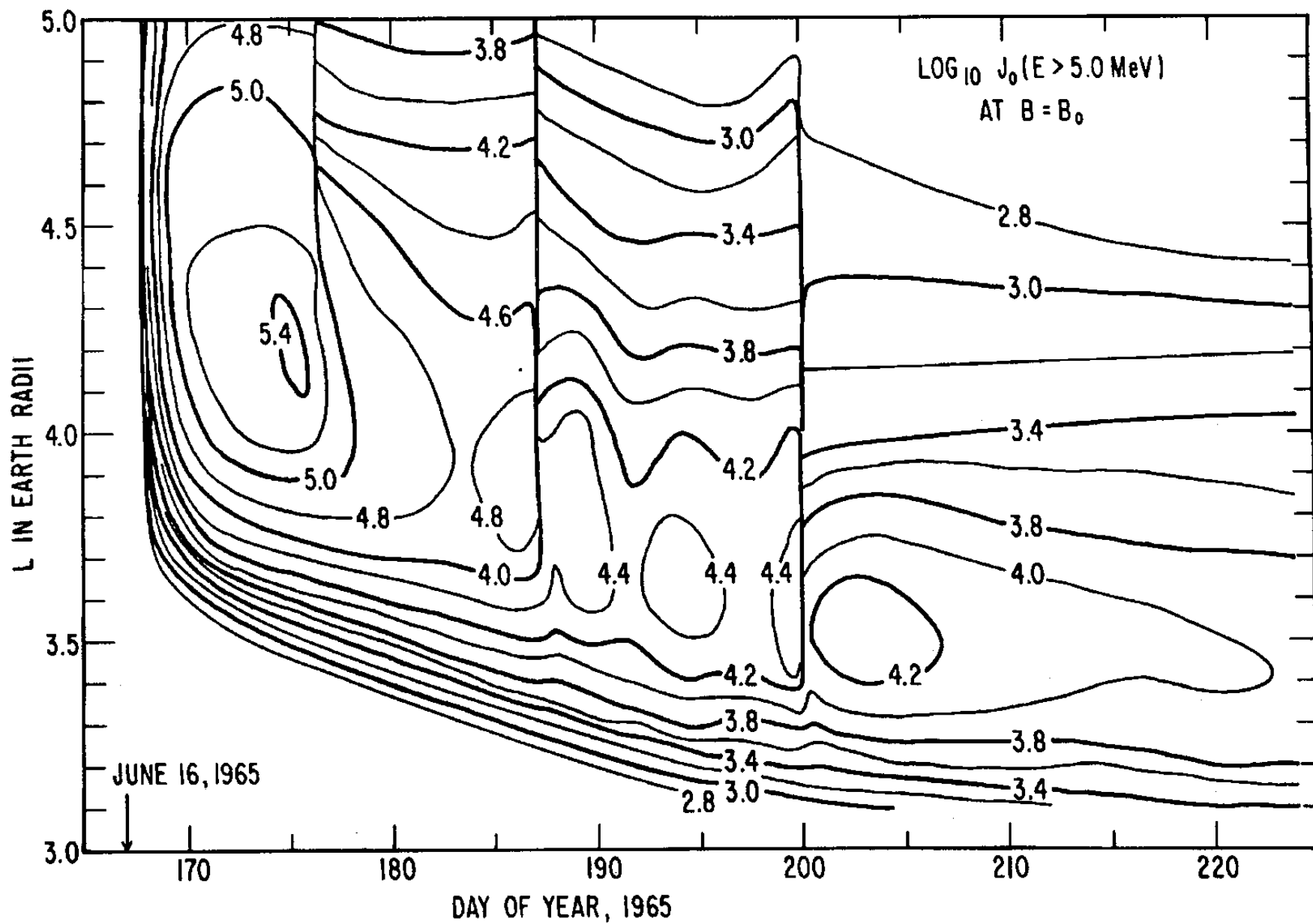
- Source of the highly variable, outer radiation belt electrons



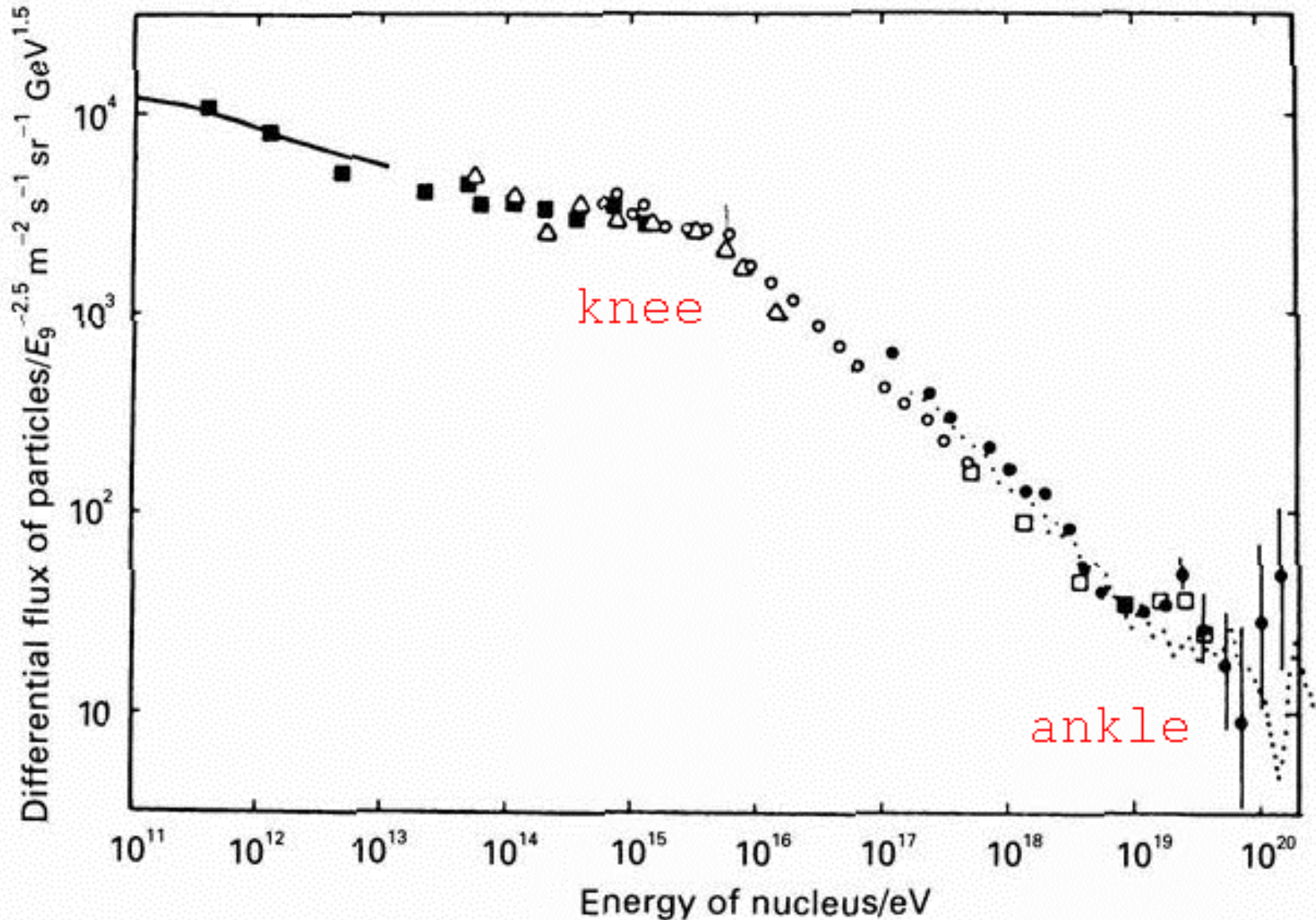
- Source of Galactic cosmic rays.



# McIlwain, 1966



# Galactic Cosmic Ray Spectrum

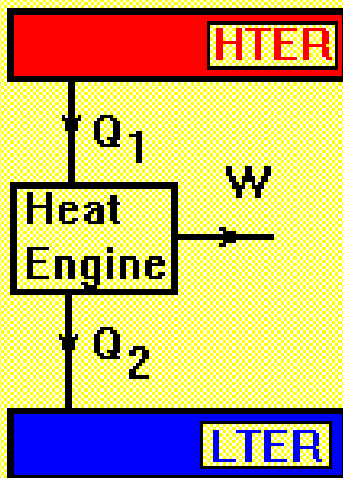


# Plasma Thermodynamics

- A neglected field—so this discussion is very qualitative. (Please give me references!)
- In ideal gases the Maxwell equilibrium is extremely well maintained by collisions
- In plasmas, turbulence takes the place of collisions. But the long-range Coulomb interaction means that equilibrium is often attained very slowly.
- Therefore one must get accustomed to non-Maxwellian distributions, and non-standard measures of temperature and entropy. E.g.  $k_{\text{app}}$

# What is Acceleration?

- Entropy is often defined as  $S = Q/T$ . Thus highly energetic particles have low entropy. Often they are “bump-on-a-tail” distributions, or “power law” greater than Maxwellian. Such low-entropy conditions cannot happen spontaneously, so some other process must increase in entropy. Separating the two mechanisms, we have a classic heat-engine.



Note: the entropy INCREASE of heat flow into the cold reservoir, is counterbalanced by the DECREASE of entropy into the Work.



# Efficiency of Acceleration

- The efficiency of a heat engine is easy to describe:  $e=W/Q_1$ . The work produced for some amount of heat. We can also use a slightly modified definition by dividing by temperature,  $T$ :  $e=S_w/S_Q$
- We can now use these definitions to categorize the various acceleration mechanisms that have been proposed for radiation belts and cosmic rays.

– Waves	Very Low	$< 10^{-4}$
– Reconnection	Low	$< 10^{-3}$
– Fermi I and II	Medium	$< 10^{-1}$
– Shocks	Low	$< 10^{-3}$

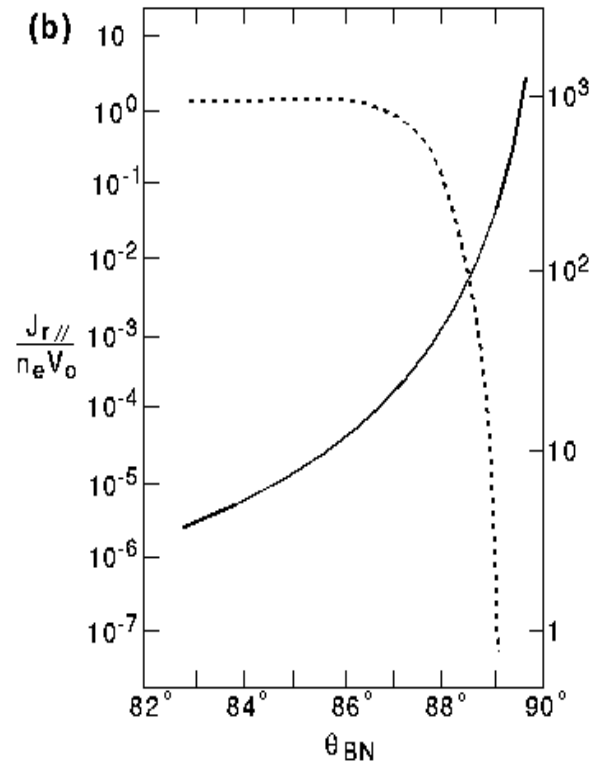
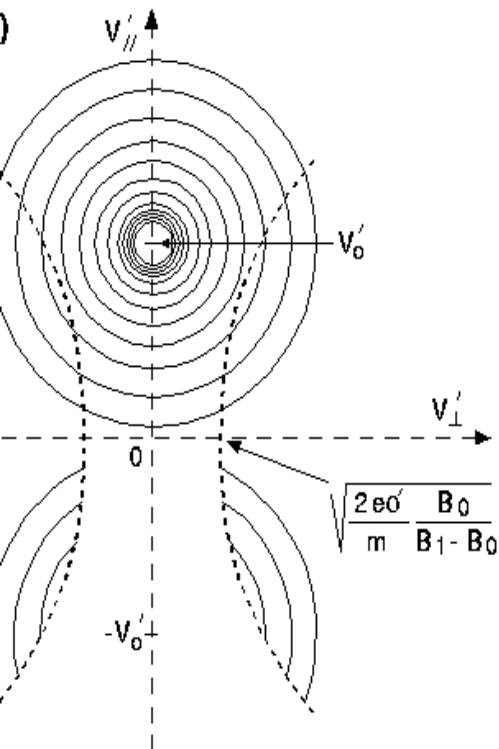
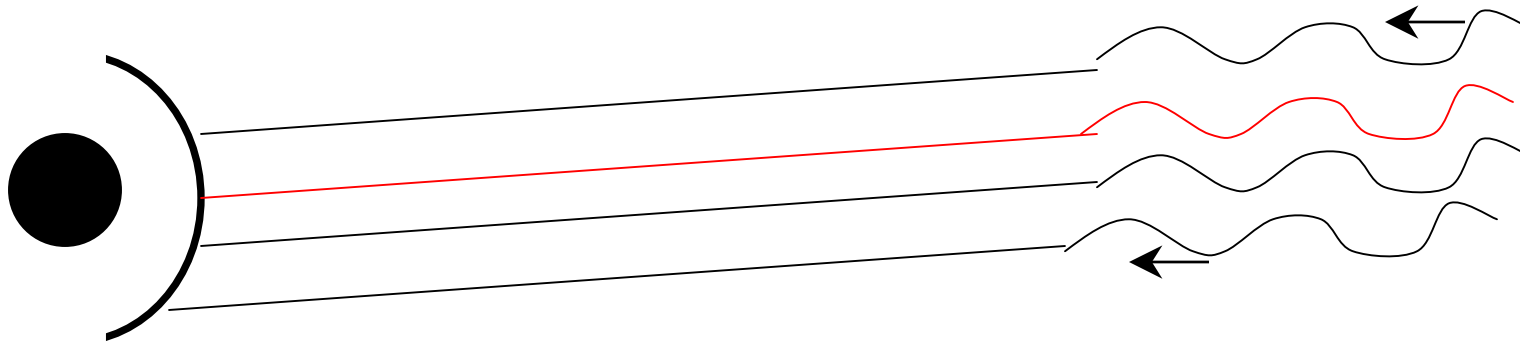
# Why does a Trap help?

- Single-stage acceleration mechanisms operate very far from equilibrium. They therefore have a huge difference in entropy between W and Q, and are therefore inherently inefficient. Likewise environmental conditions are far from equilibrium, and are thus inherently unlikely. The product of these two situations = low density of accelerated particles.

**Energy Source \* Efficiency \* Probability = Power**

- A trap allows small impulses to be cumulative. The pulses are close to equilibrium and are also likely. Thus a trap = higher density of accelerated particles.

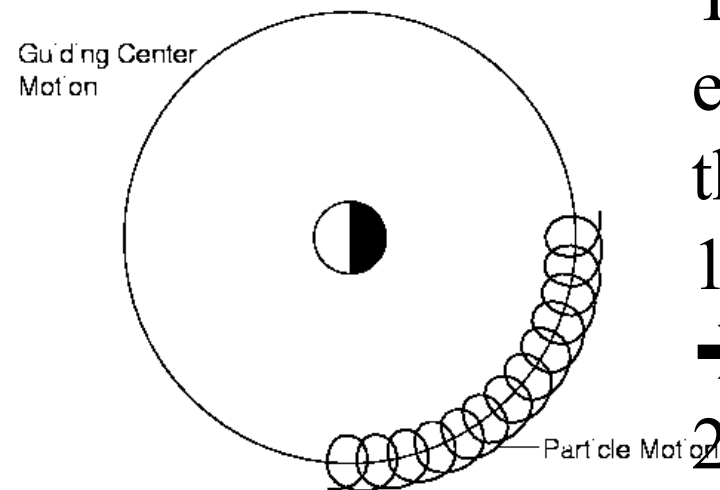
# The Fermi-Trap Accelerator



Waves convecting with the solar wind, compress trapped ions between the local  $|B|$  enhancement and the planetary bow shock, resulting in 1-D compression, or  $E_{\parallel}$  enhancement.

# The Dipole Trap

- The dipole trap has a positive B-gradient that causes particles to trap, by  $\mathbf{E} \times \mathbf{B}$ -drift in the equatorial plane.



There are 3 symmetries to the Dipole each of which has its own “constant of the motion” (Hamiltonian is periodic).

1) Gyromotion around B-field

→ Magnetic moment, “O”;

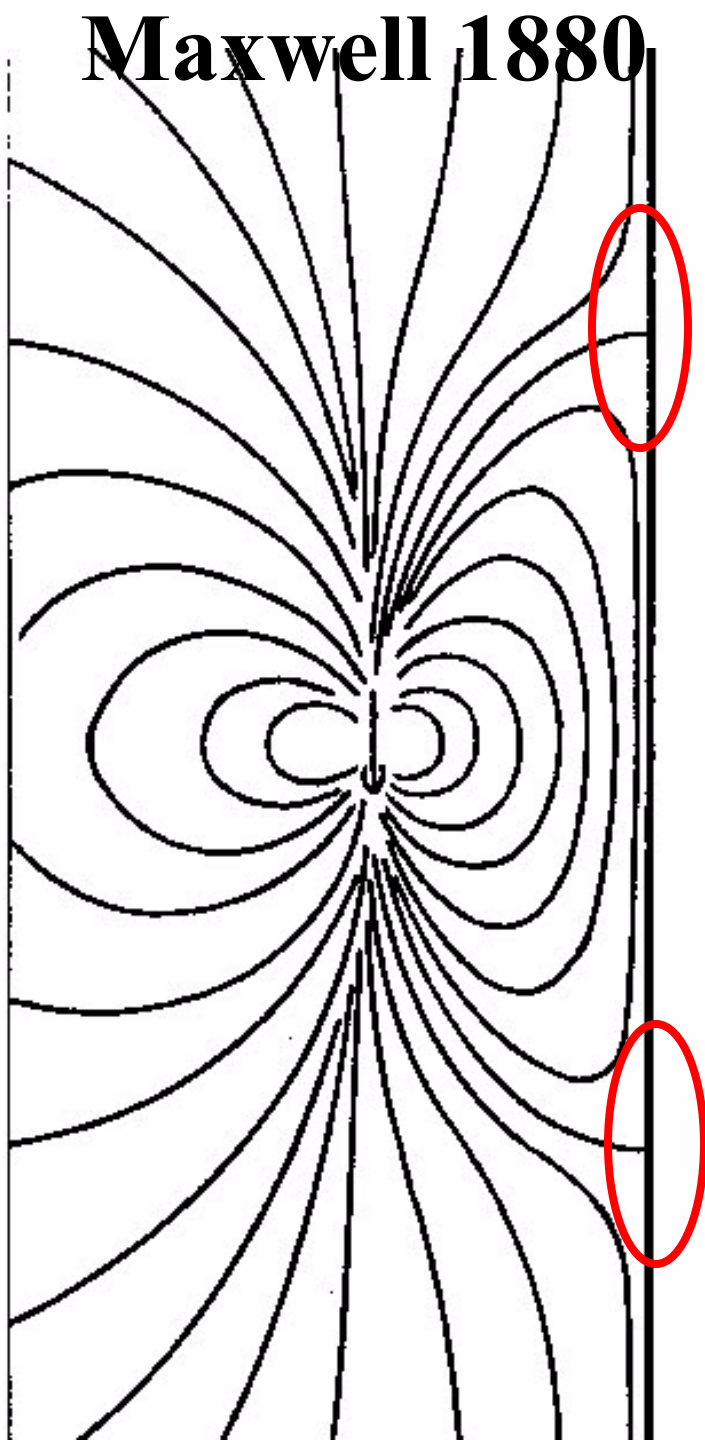
2) Reflection symmetry about equator → Bounce invariant “J”;

3) Cylindrical symmetry about z-axis → Drift invariant “L”

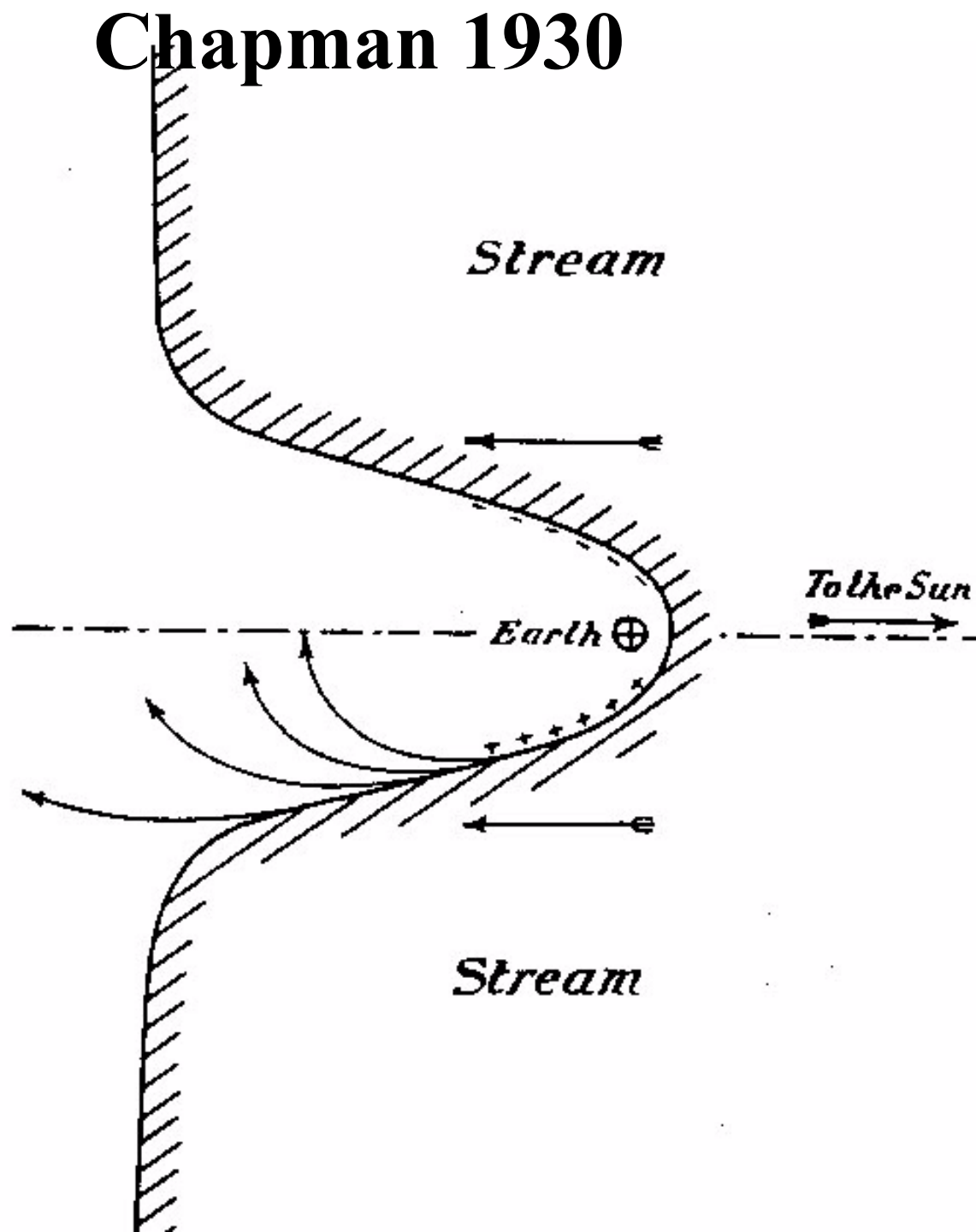
# The Quadrupole Trap

- A quadrupole is simply the sum of two dipoles.
  - Dipole moving through a magnetized plasma, heliosphere, magnetosphere, galactosphere
  - A binary system of magnetized objects –binary stars
  - A distributed current system-Earth's core, supernovae
- Quadrupoles have “null-points” which stably trap charged particles (eg. Paul trap used in atomic physics)
- Maxwell showed that a perfect conducting plane would “reflect” a dipole & form a quadrupole

Maxwell 1880

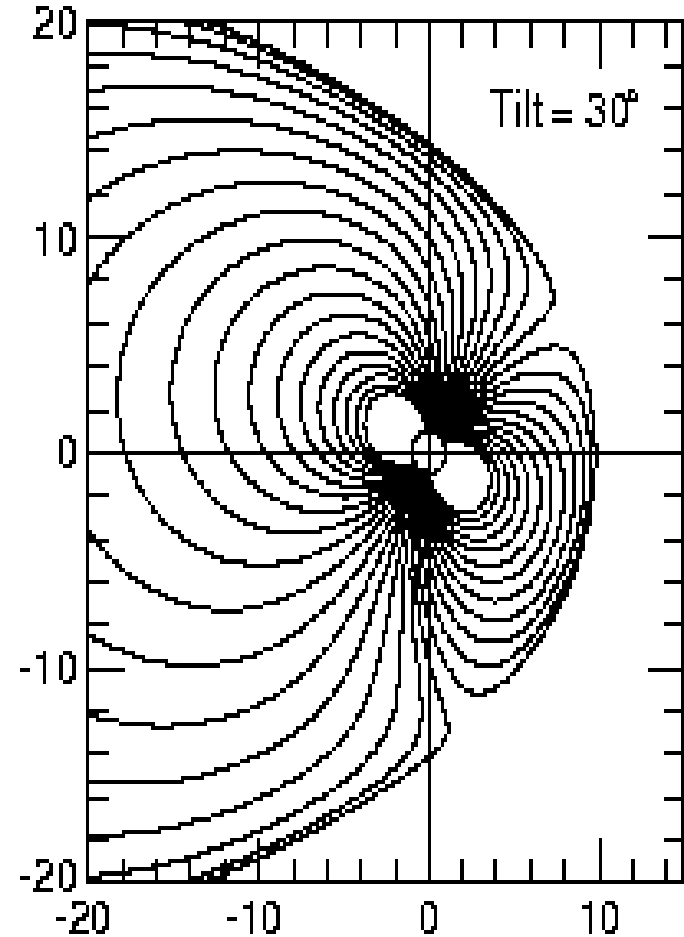
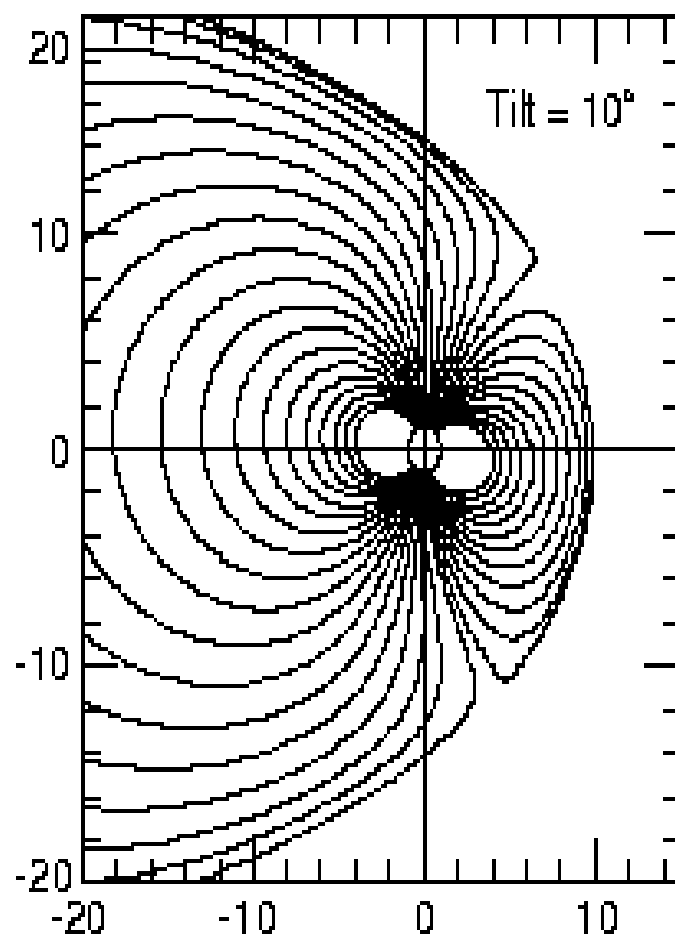
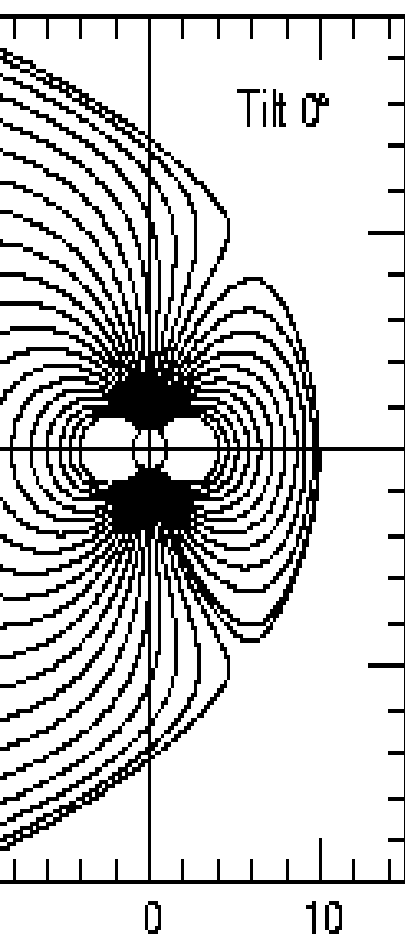


Chapman 1930



# Tsyganenko Cusps

Tsyganenko Vacuum Magnetosphere



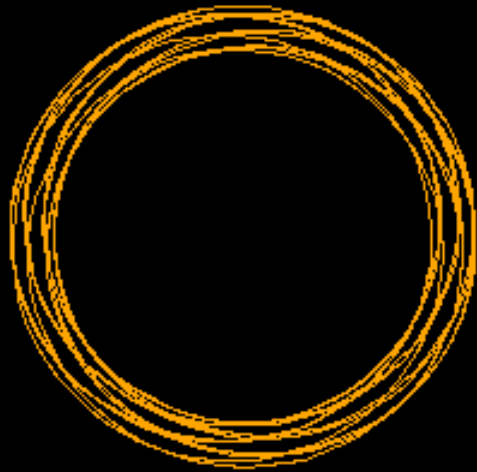
$X (R_E)$

# Why is the Dipole a better trap than Accelerator?

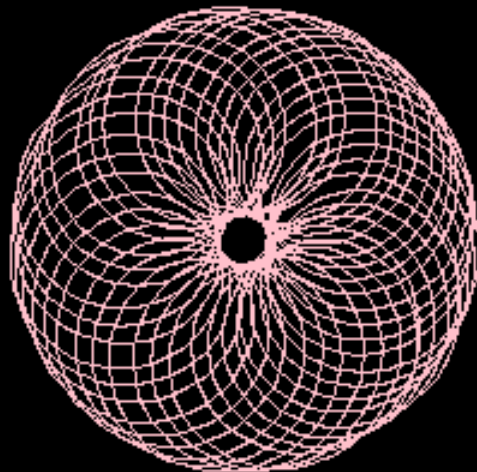
- Three “adiabatic invariants” ( $\mathcal{O}$ ,  $J$ ,  $L$ ) w/characteristic times of 1ms, 1s, 1000s
- KAM theorem: the 3 adiabatic invariants are separated by factors of 1000 in time, and therefore do not “mix”. So dynamical evolution, or diffusive/stochastic “acceleration” is not fast.
- Arnol’d diffusion, or chaotic motion in the Poincaré section, can lead to rapid diffusion if these “basins” are connected to each other--“Arnol’d web”



# Kolmogorov, Arnol'd, Moser Theorem

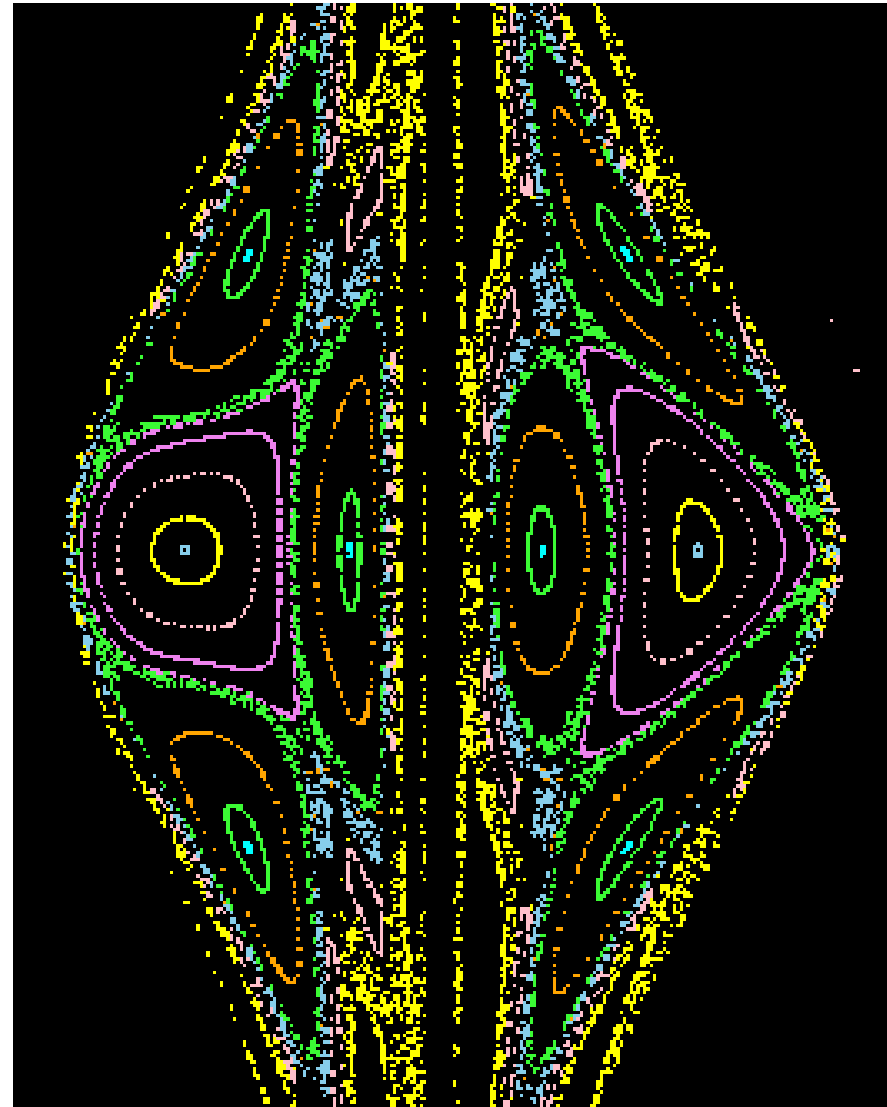


Earth orbit as  
Perturbed by  
Jupiter.



Poincaré slice  
 $x$  vs.  $v_x$  taken  
along the E-J  
line.

Earth orbit if  
Jupiter were  
50k Earth  
masses.

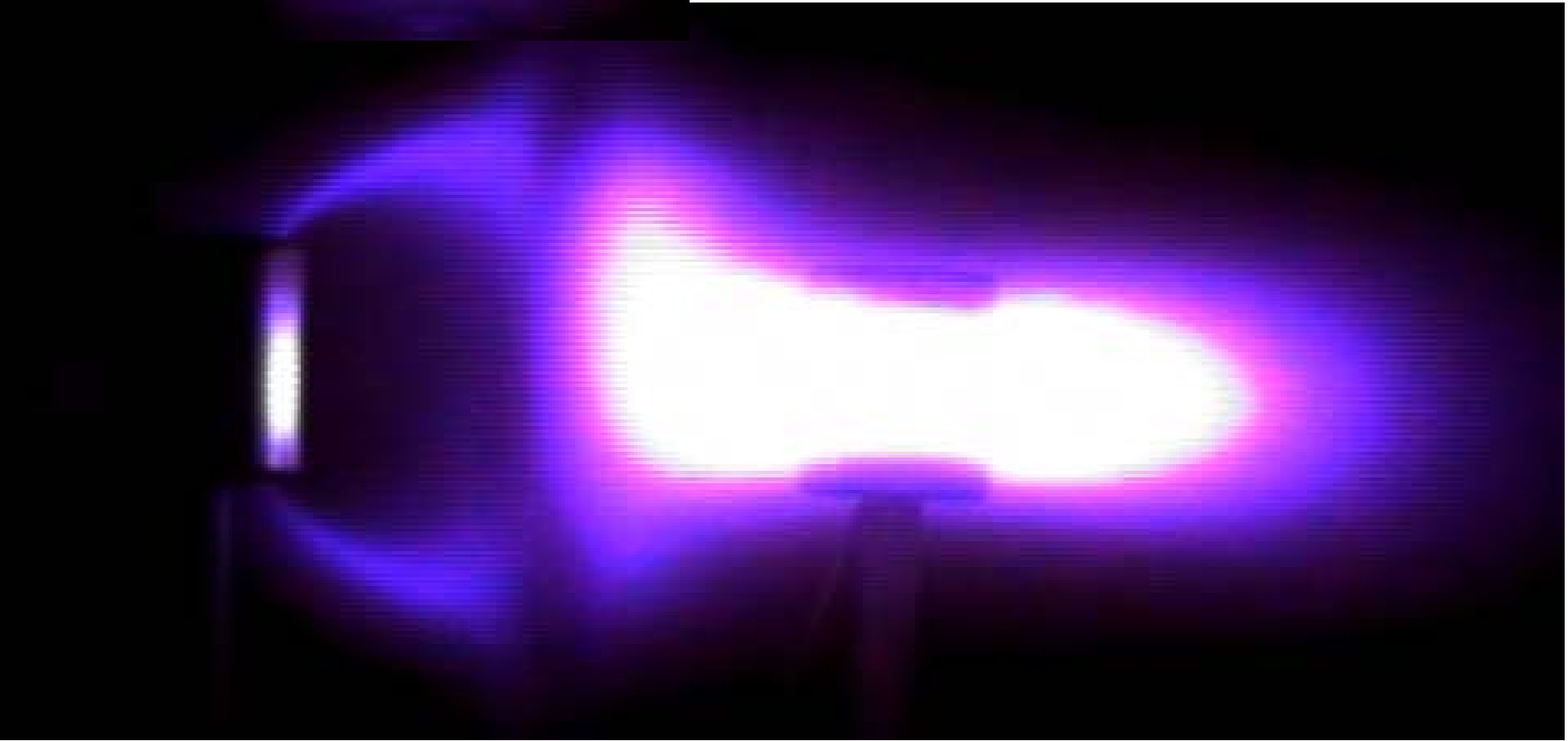
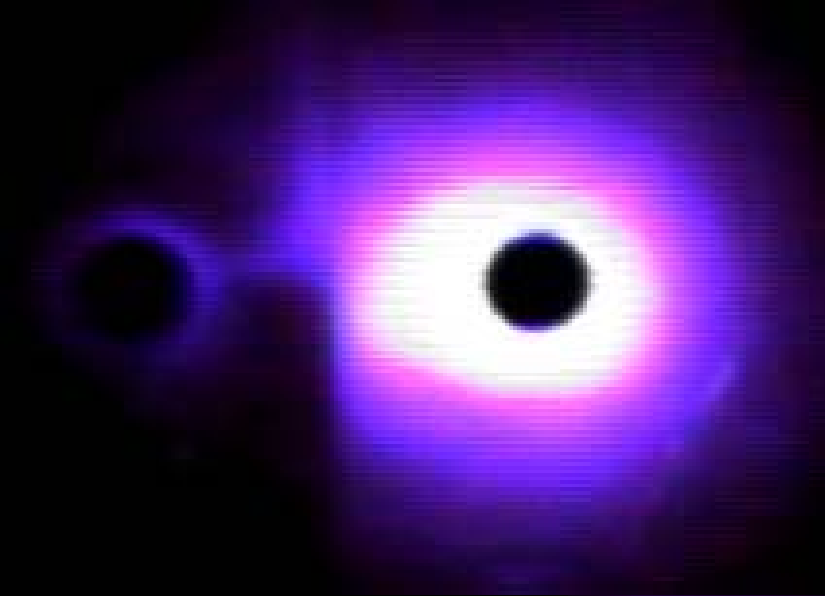


# Comparison of Dipole & Quadrupole

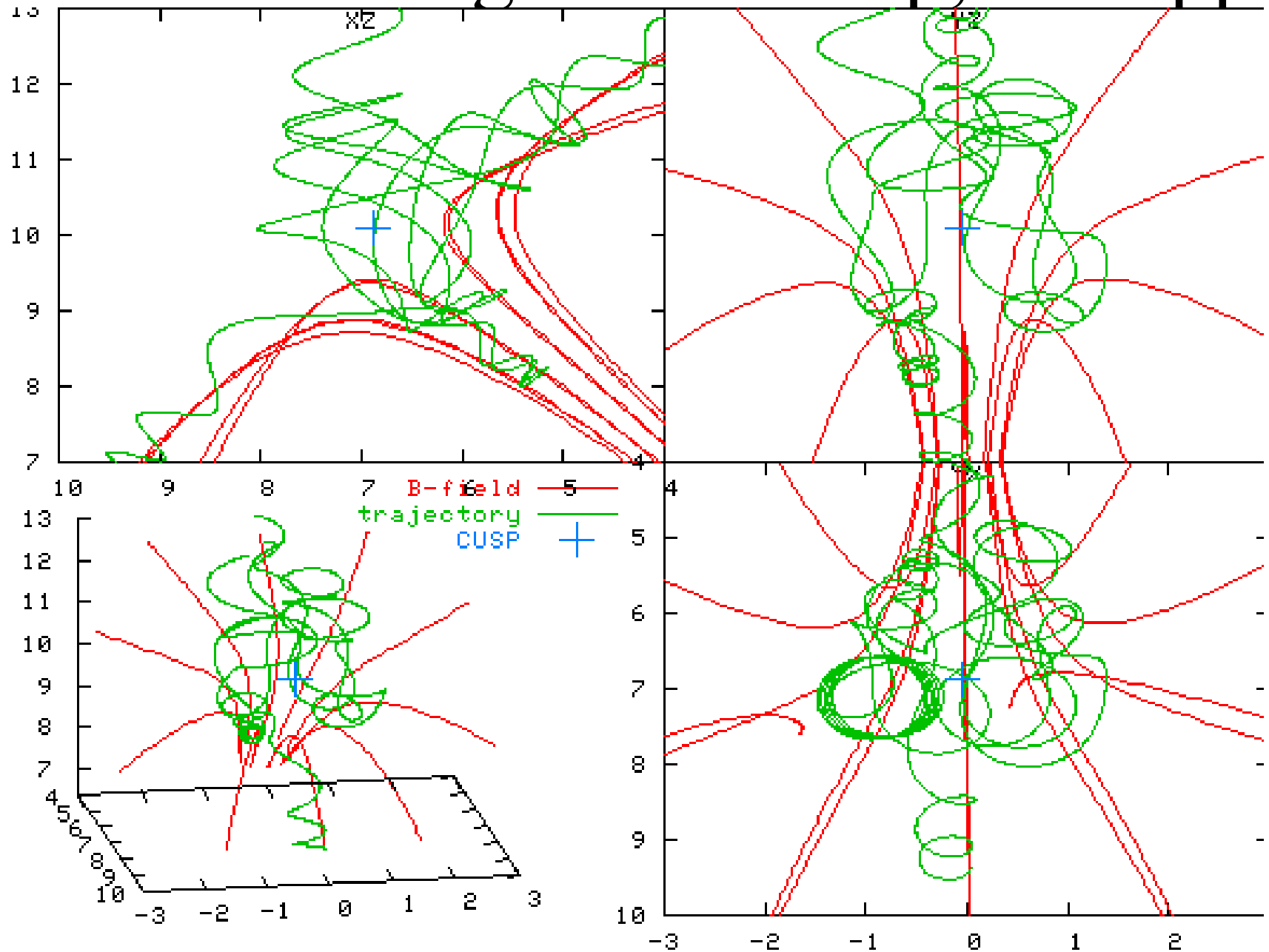
- Rim fed, center exit
  - Exit blocked by magnet
  - Compressions mostly affect the low-energy part
  - High-E particles are better trapped than Low-E
  - Adiabatic invariant times separated by factor  $\sim 1000$
  - Scattering (diffusion) out of the trap is difficult and de-energizing / deaccelerating.
- Center fed, rim exit
  - Exit accessible
  - Compressions affect whole distribution
  - High-E particles less trapped than Low-E
  - Adiabatic invariant times separated by factor  $\sim 10$
  - Scattering out of trap is easy and may also energize/ accelerate

# Quadrupole Trap in the Laboratory

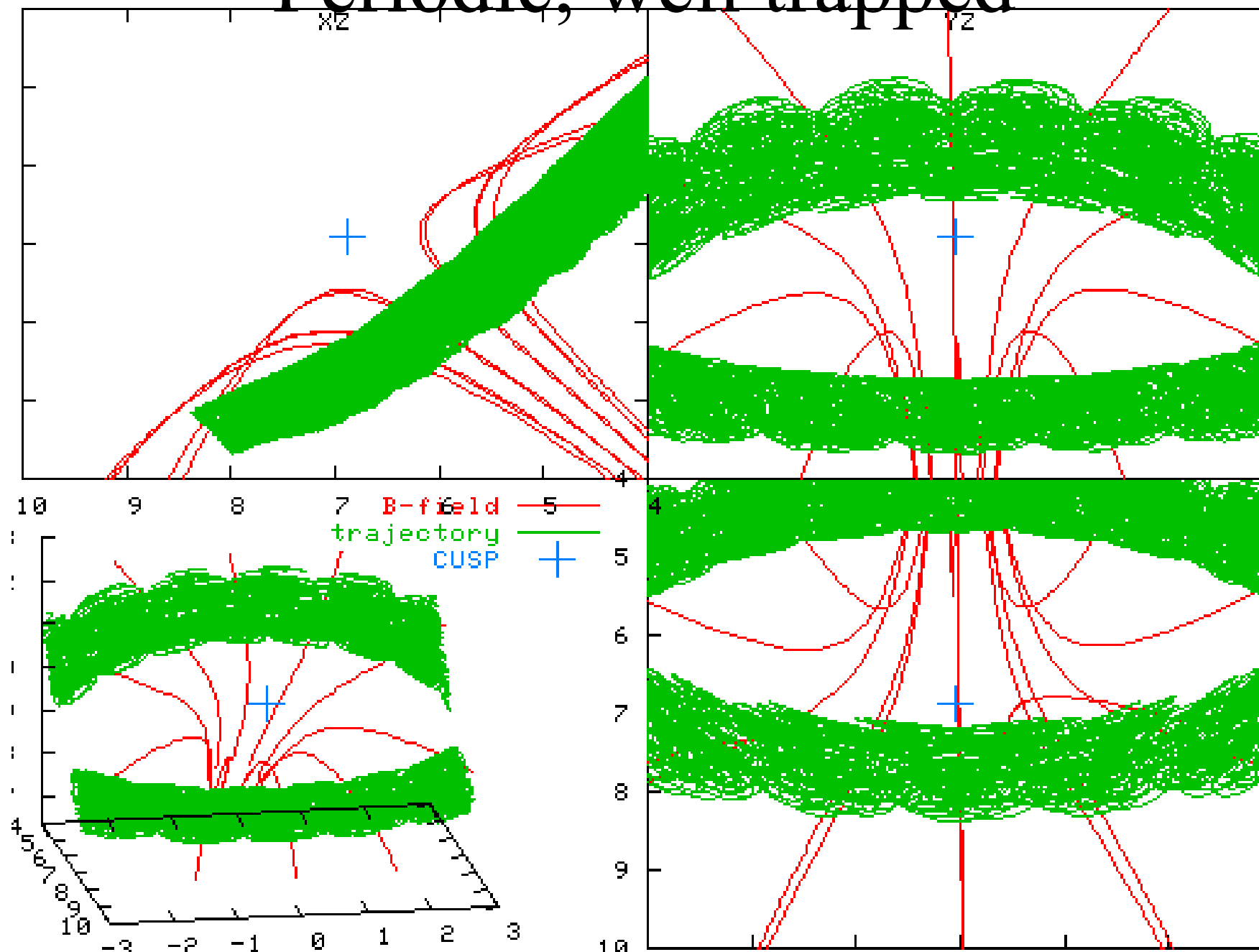
(Two 1-T magnets, -400V,  
50mTorr)



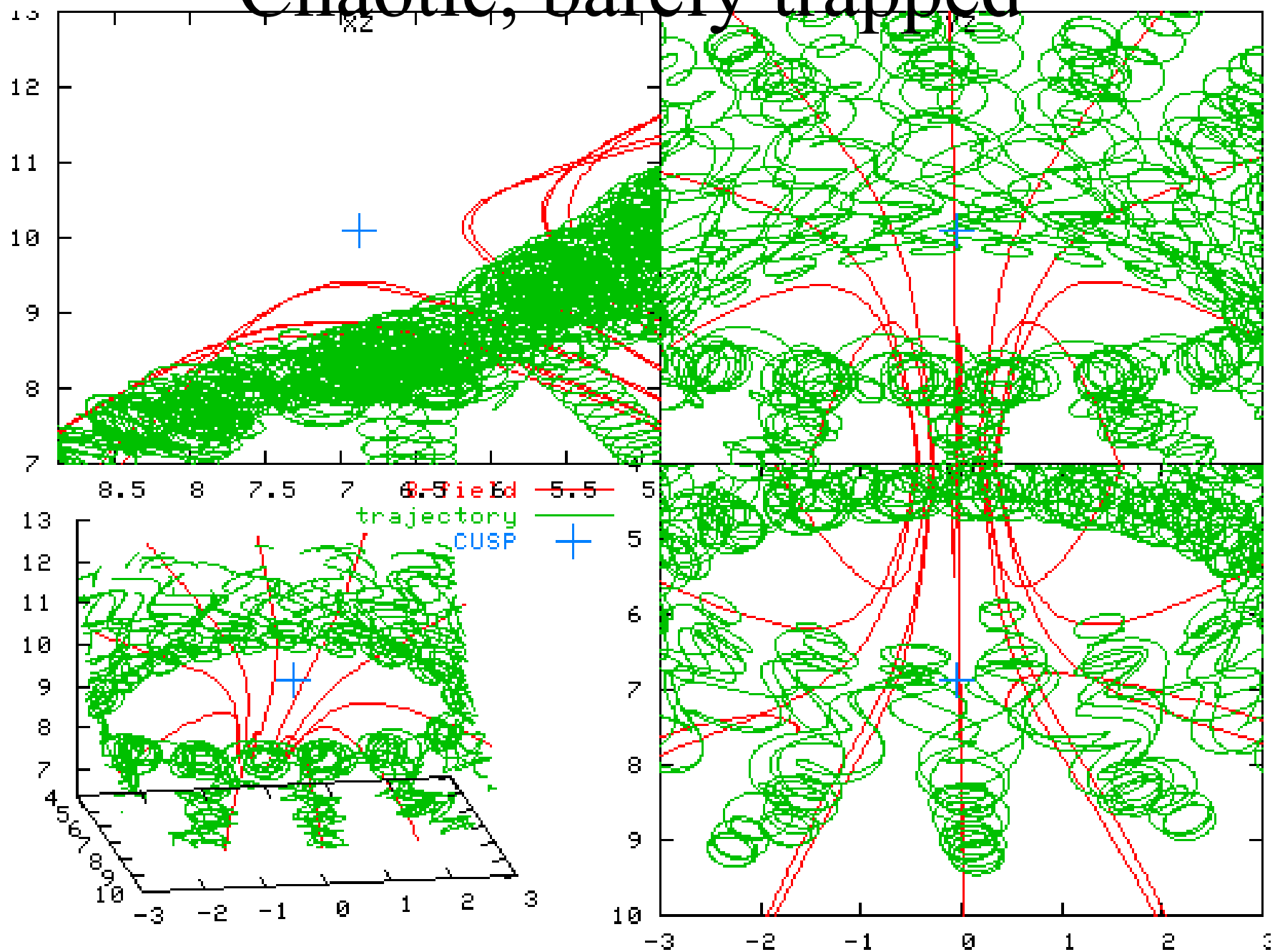
# Particle Tracing in T96 cusp, untrapped



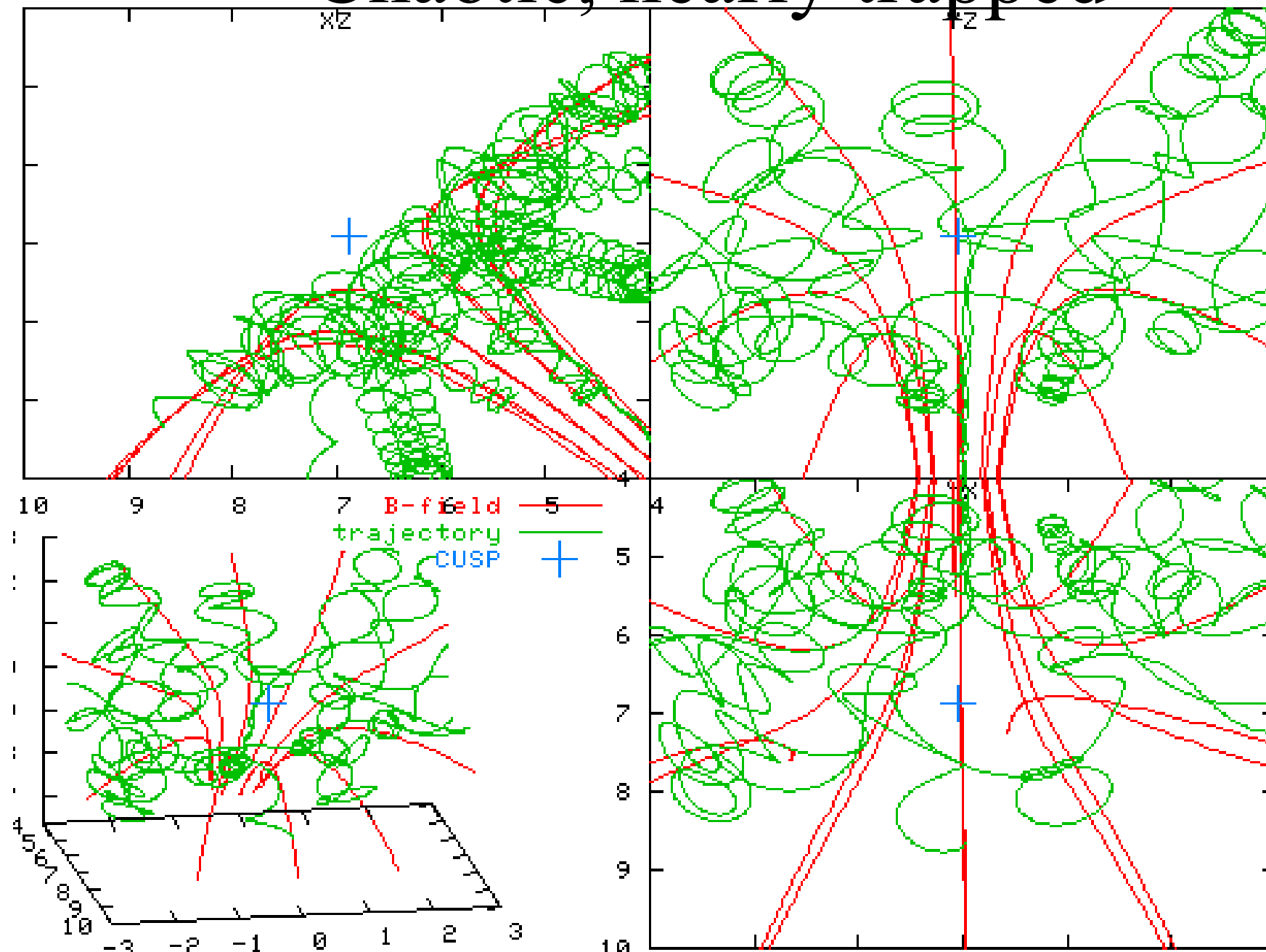
# Periodic, well trapped



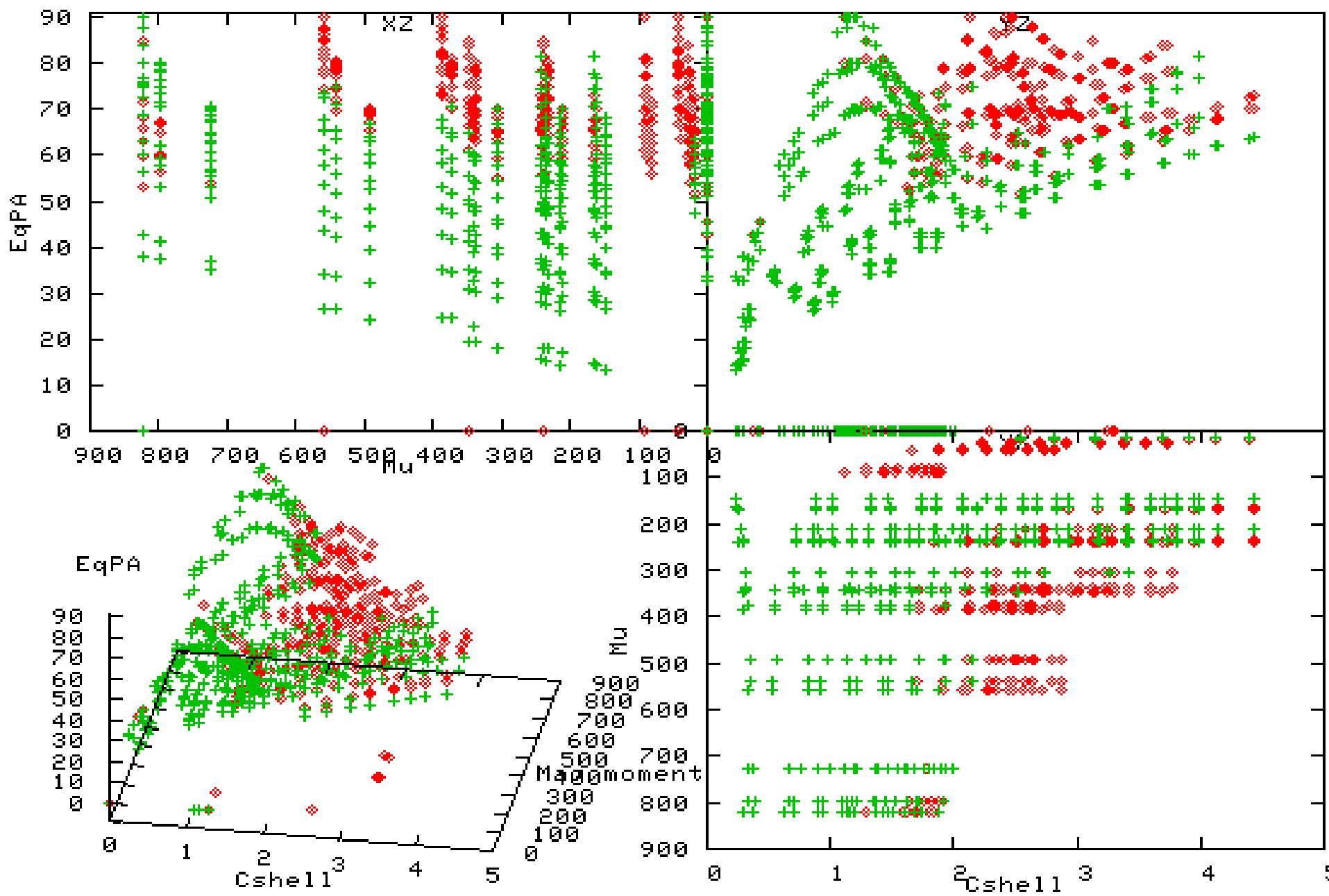
# Chaotic, barely trapped



# Chaotic, nearly trapped

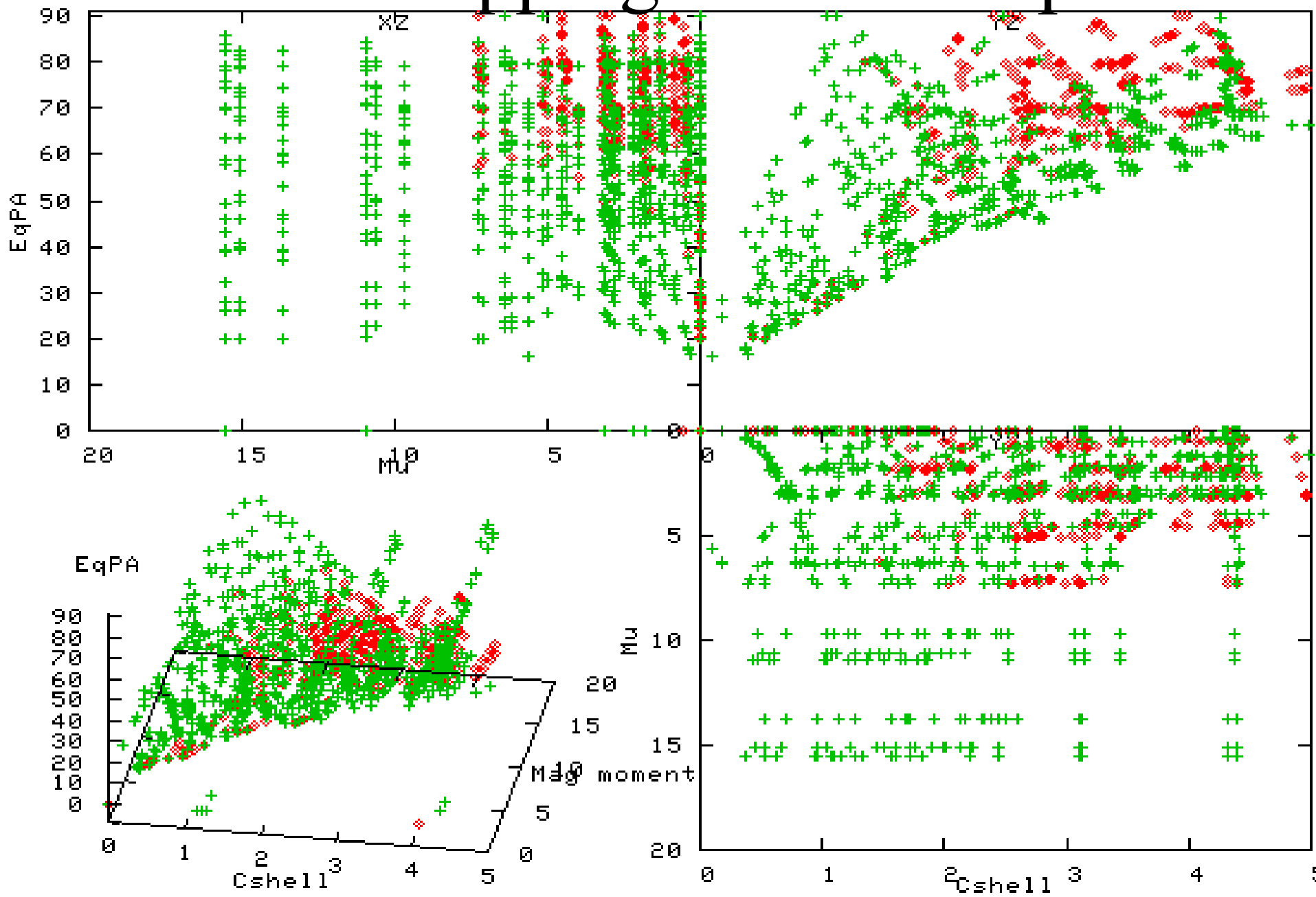


# e- Trapping Regions of T96 Cusp





# H<sup>+</sup> Trapping in T96 Cusp



# Poincaré Plots?

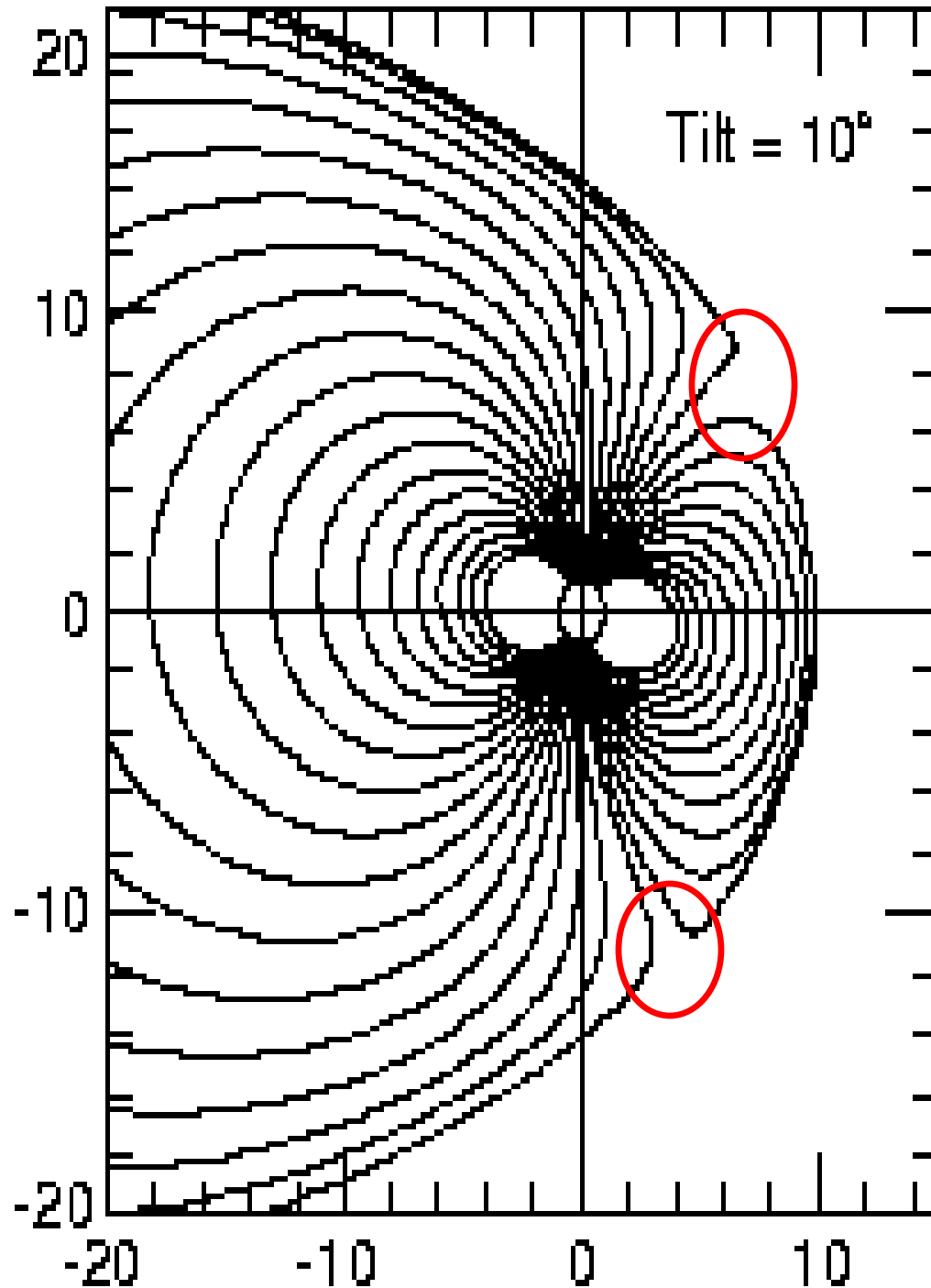
- Now that we know there are trapping regions of the cusp, e.g. periodic orbits, we can display those orbits with a Poincaré section. Then we can potentially demonstrate the invariant tori and chaotic orbits.
- Unfortunately, I only realized the advantage of this approach after plotting 4000 trajectories  $T < \text{drift-time}$  and visually classifying them as “untrapped”, “trapped periodic”, and “trapped chaotic”. (Using 4 CPU weeks on dual AMD 1.8GHz)
- So the Poincaré plot (many drift orbits) will have to wait for CPU time.

# How does the Quadrupole Accelerate?

- Mathematically, we say that the basins of chaos are interconnected, not constrained by invariant tori. They are connected in an Arnold's web that allows rapid stochastic acceleration.
- More prosaically, compressions adiabatically increase the particle energy, while the center of the trap acts as a “mixmaster” for redistributing that energy into other invariants. Consequently particles diffuse in energy very rapidly.
- A comparison to Fermi I,II is highly instructive for understanding the mechanism.

# Acceleration via Random Impulses

- Fermi-I acceleration can be thought of as a 1-D compression of a magnetized plasma. It heats only in the  $E_{\parallel}$  direction. Eventually the pitch angle gets too small to be reflected by the upstream waves, and it escapes. If pitch angle diffusion occurs, a particle may convert  $E_{\parallel} \rightarrow E_{\perp}$ , and continue to gain energy.
- The waves, or compressive impulses need not be coherent or particularly large, especially if scattering is occurring. Thus the entropy of the energy source is relatively high, the probability is high = high efficiency acceleration. Observations support this.



# Cusp Orientation

When the cusp points toward the sun, it “opens”, when it points away, it “closes”. Likewise, SW pressure pulses will shrink the cusp as a whole, e.g. radially compressing it.

# Fermi I,II

vs

# Alfven I,II

- 1-D compression,  $E_{\parallel}$
  - Upstream Alfven waves impinging on barrier
  - Scattering inside trap due to waves
  - Max Energy due to scale size of barrier (curvature) becoming  $\sim$  gyroradius
  - Acceleration time exponentially increasing
  - Critically depends on angle of SW B-field with shock
- 2-D compression,  $E_{\perp}$
  - Compressional waves impinging on cusp
  - Scattering inside trap due to quadrupole null point
  - Max energy due to gyroradius larger than cusp radius (rigidity).
  - Acceleration time exponentially increasing
  - Critically depends on cusp angle with SW

# Signatures of Cusp Acceleration

- Gyroradius effects:  $r = \gamma_0(mE)^{1/2}/qB$ . For a given topology, lighter masses will have higher cutoff energies.
- Energy increase is exponentially increasing function of time. (like Fermi)
- Scale size effects: the larger the cusp, and the larger B-field, the higher the cutoff E.
- Output spectra have power law tails (not bump-on-tail nor exponential maxwellians.)
- Given continuous input, output is also continuous.

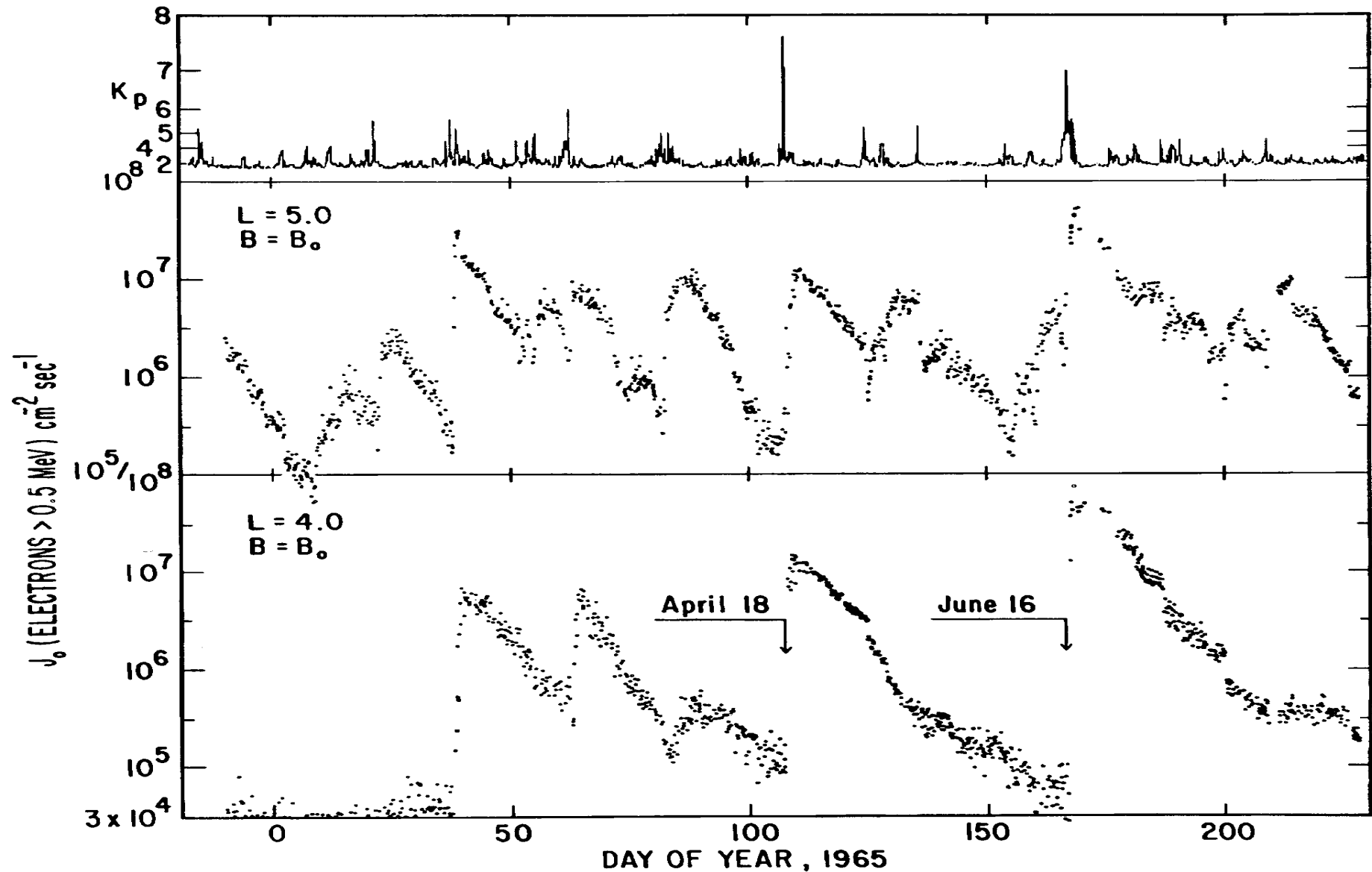
# Problem #1: Source of ORBE



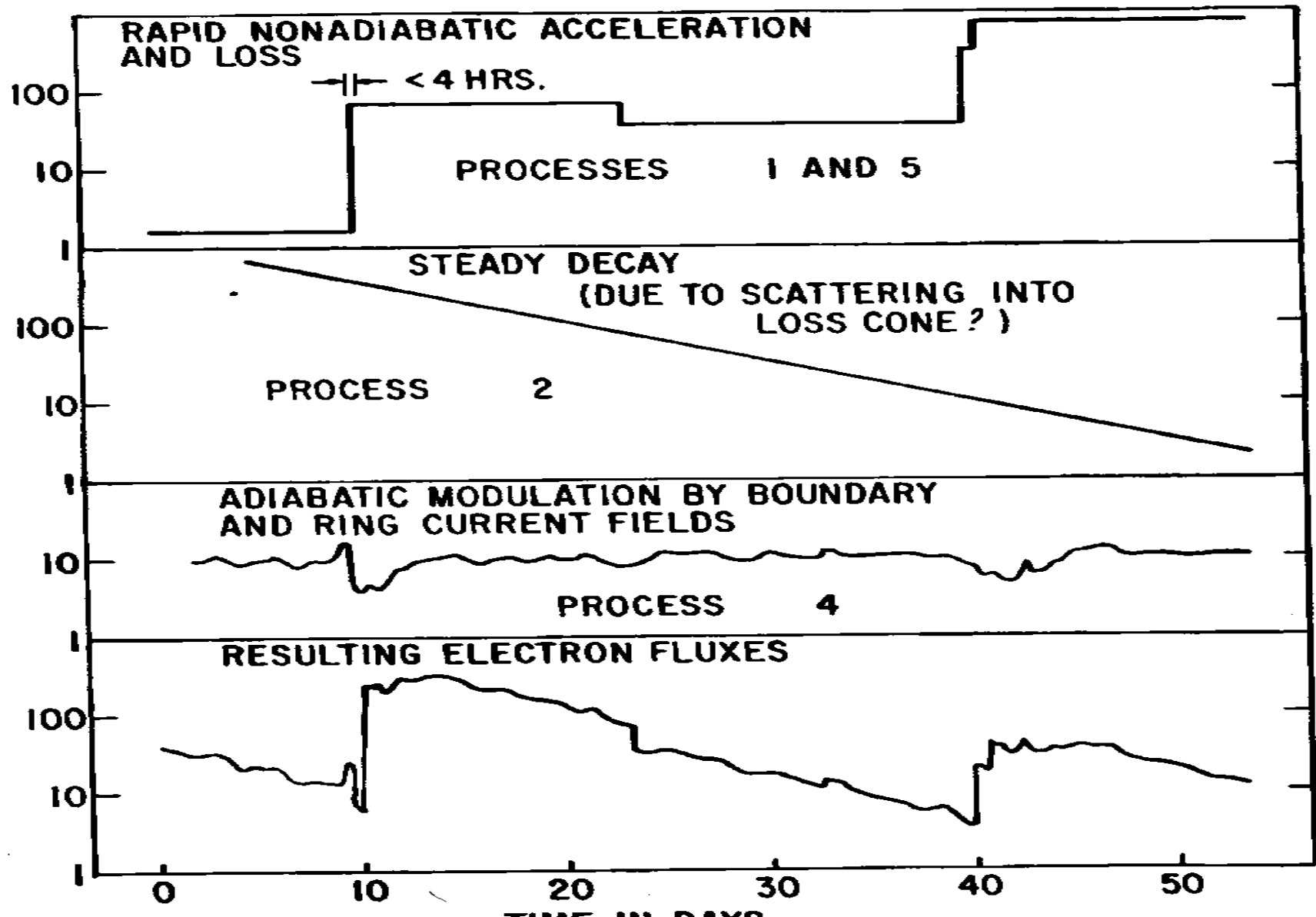
# Prediction Problems

- The hazards of MeV electrons are greatest in the outer radiation belts ( $L \sim 3-5$ ), though they still exist for LEO orbits and the South Atlantic Anomaly.
- MeV electrons penetrate, causing deep dielectric discharges, such as the Telstar 401 satellite loss.
- The Geosynchronous orbit uniquely lies on the edge of a steep spatial and spectral gradient. Thus GEO poses a wildly variable MeV electron environment, which is nearly impossible to predict, both in principle and in practice.
- Yet of all orbits, this is THE most crowded spot.

# ORBE (McIlwain 1966)



# McIlwain 1966



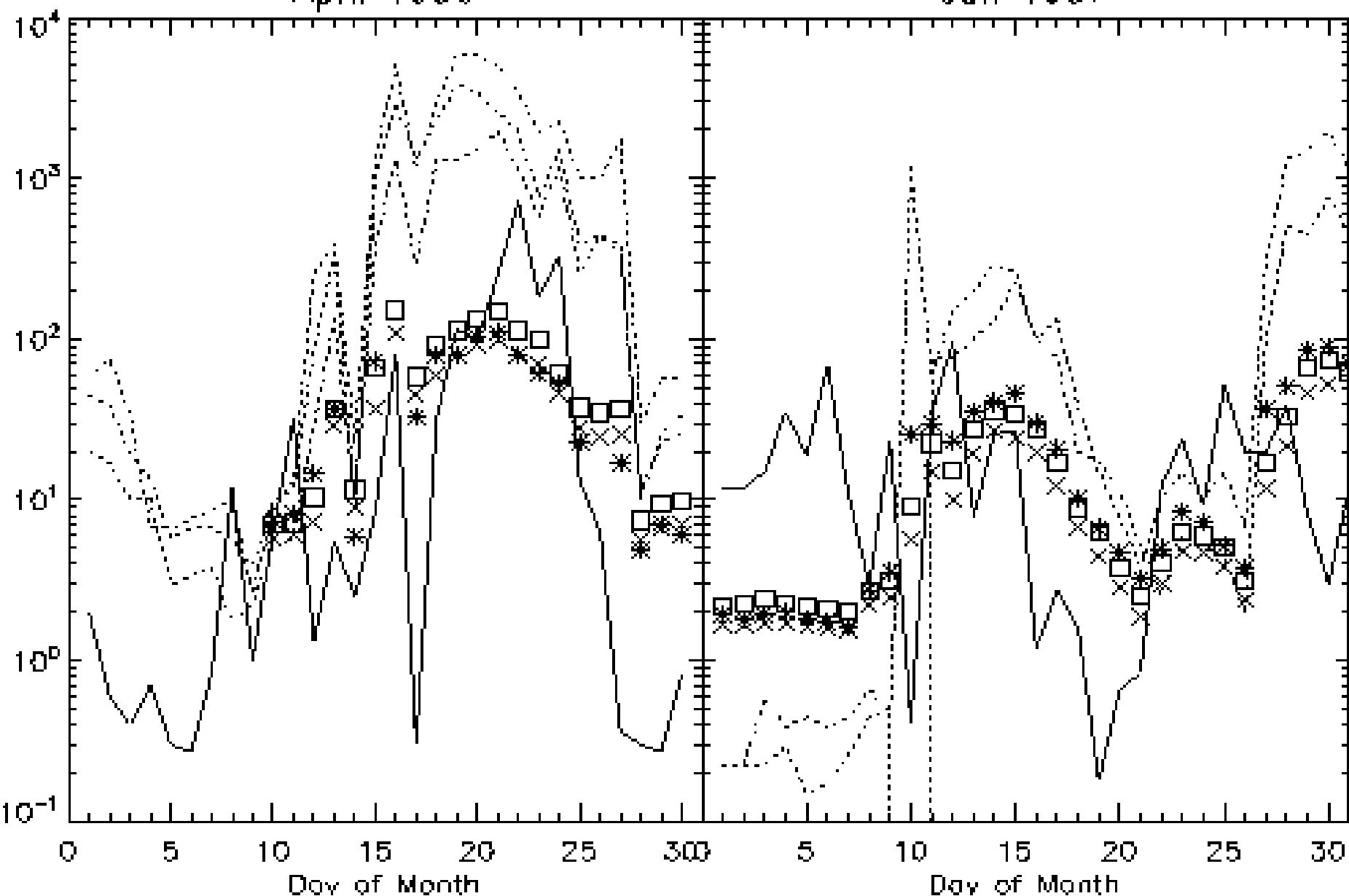
# Details of Injection (#1&5)

- They have a 24-48 hour typical rise time from a SW disturbance (shock, Dst storm, etc.) But can be as short as 8 hr (Jan 97) or as long as 72 hours.
- The intensity roughly follows a solar cycle dependence but can vary by 3 orders of magnitude
- The spectral hardness generally increases during the storm, but exhibits a “knee” that whose cutoff strongly depends on the unpredictable magnitude
- Best correlation with  $V_{sw}$  (70%) but  $B_z$  or  $n_{sw}$  ruins it. Neither mechanical nor electrical SW driver

# Koons & Gornev Prediction Filter

April 1996

Jan 1997



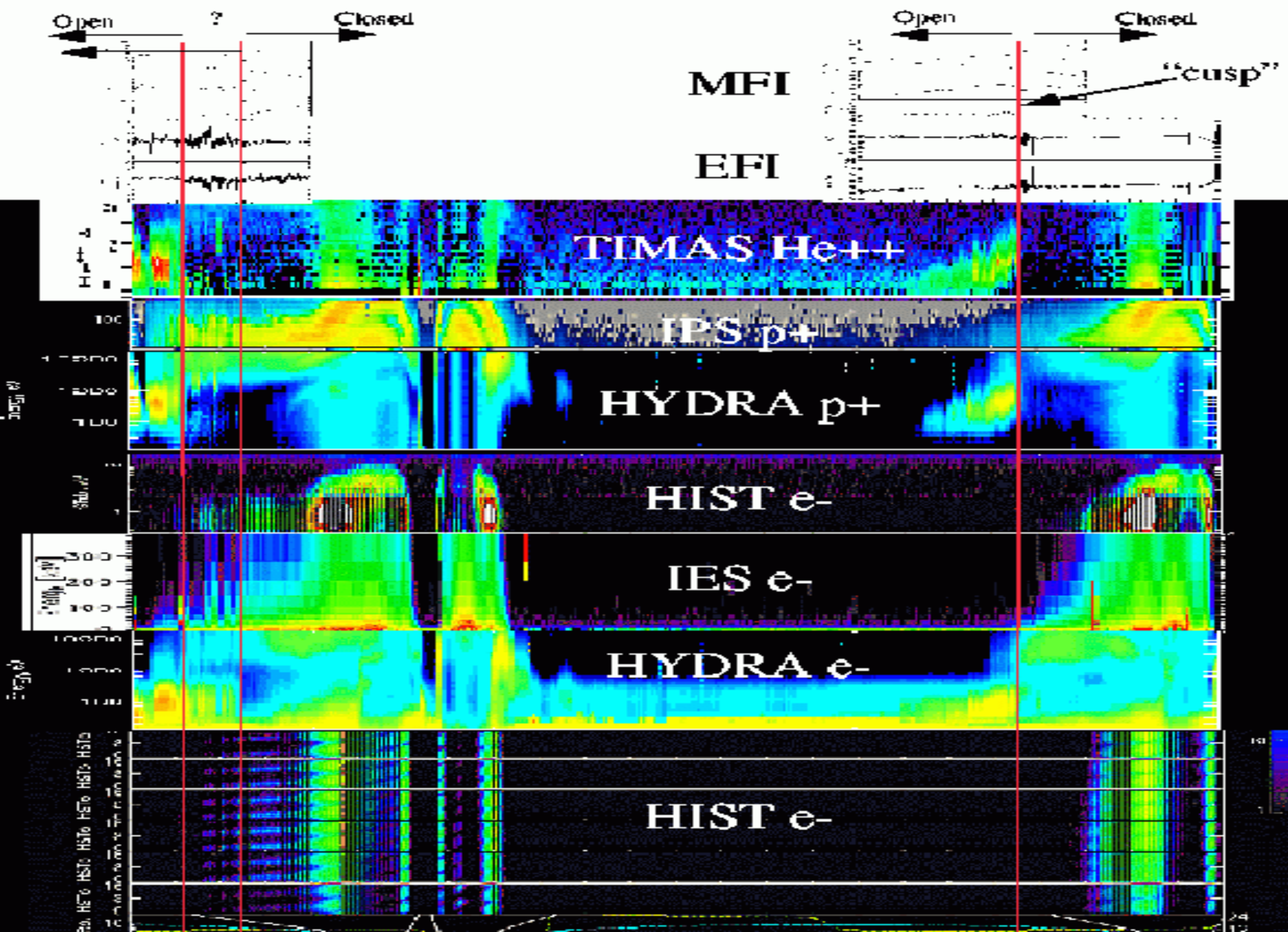
# The Prediction Paradox

- The storms we want to predict, are the **BIG** ones, but our best statistics are the *little* storms. Statistical precursors, neural nets, linear filters ALWAYS do better on the little storms, not the **killer storms**.
- We will never achieve reasonable prediction until we understand how/where/why **BIG** storms occur. Everything else is gambling.
- But the statistics have not led to better physics. Why is  $V_{sw}$  the best correlator? Especially since Dst is better correlated to  $E_y$  and AE to  $B_z$ . Why is Kp the best correlated ground-based index?

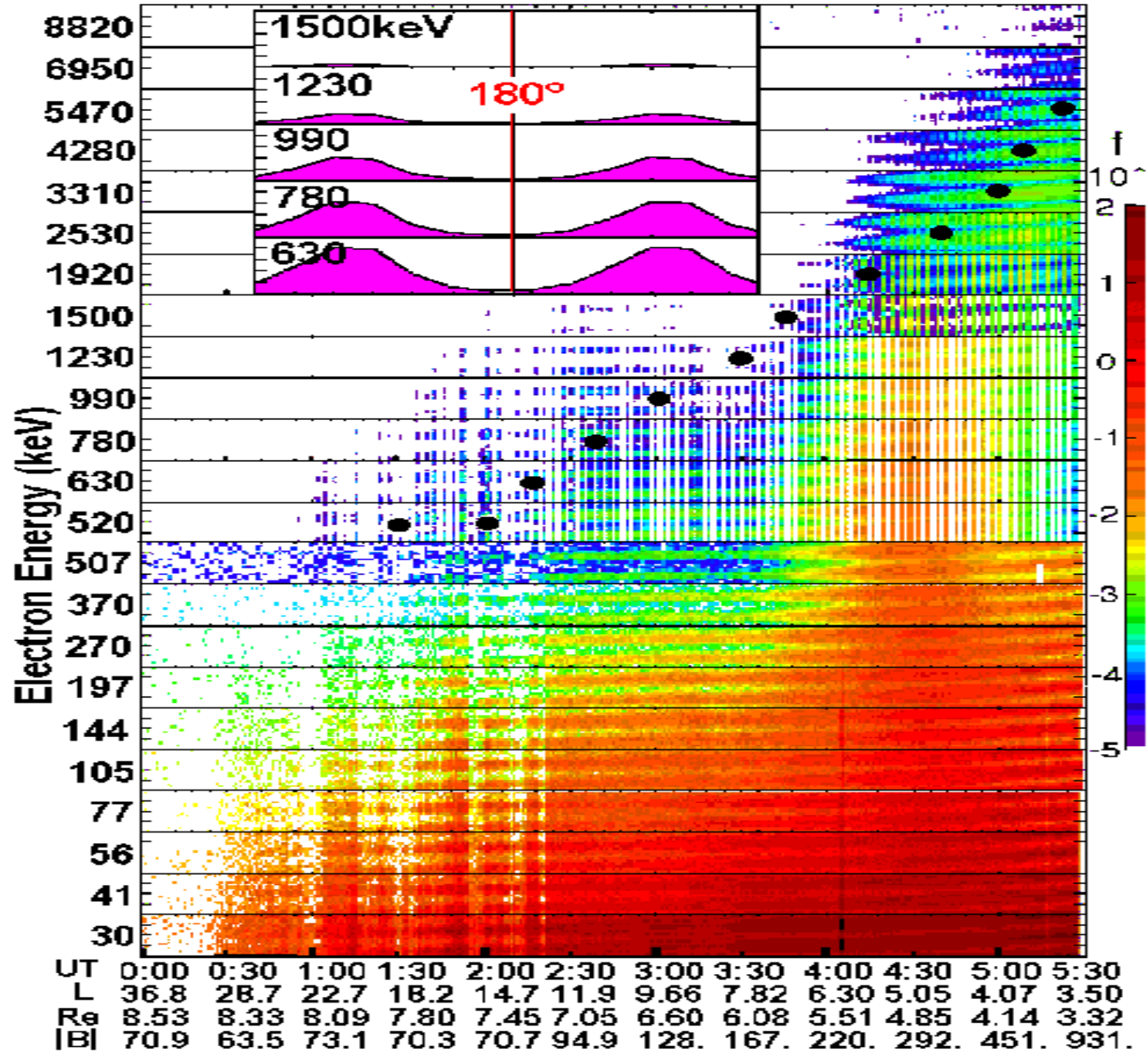
# The Argument

- 1) Electrons, not protons are injected.
- 2) Radial gradients point to an external source that is NOT the solarwind, NOR the dipole trapped belts.
- 3) Risetimes are too slow (2 days) to be 1st order acceleration, more likely 2nd order stochastic, ordered by energy dependent exponential risetimes.
- 4) Neither AE nor Dst correlate as well as Kp. Global disturbances work better than tail or RC.
- 5) MLT enhancements begin at noon.
- 6) Low altitude data are consistent with simultaneous inward radial transport & PA scattering

## POLAR Data from 10/14/96

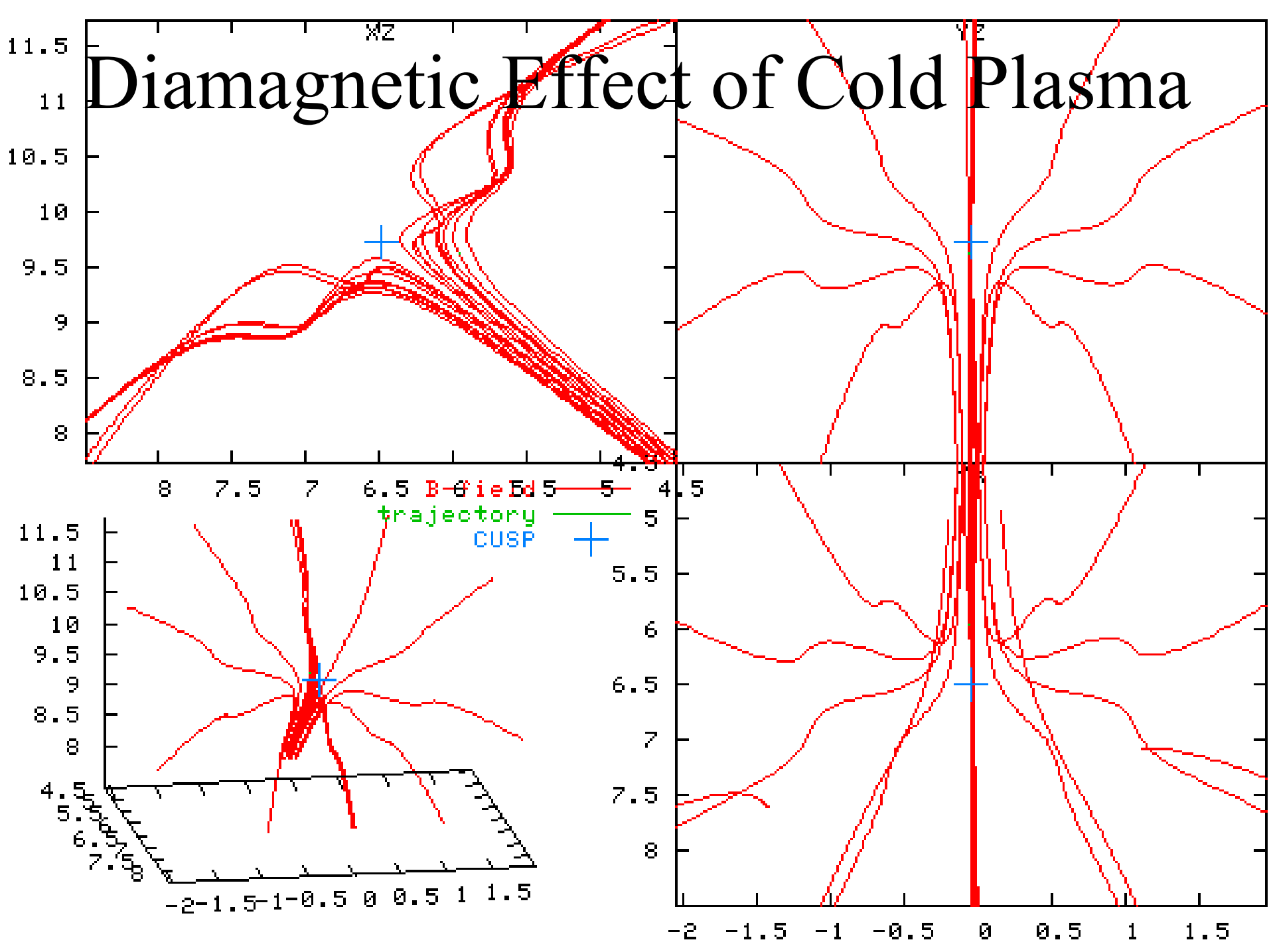




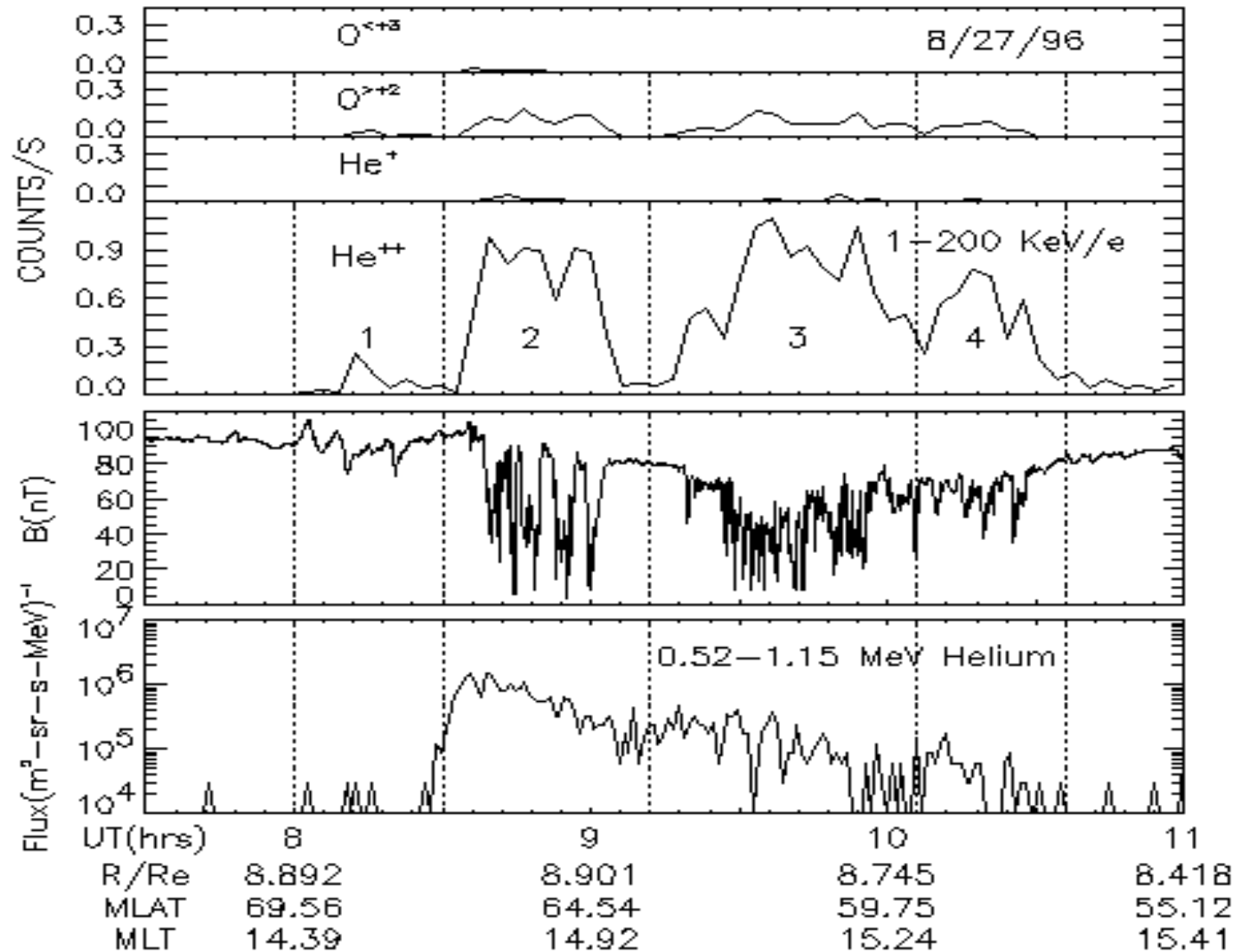


# Problems with Cusp Acceleration

- We said that Cusp acceleration was continuous, given continuous injection. How then do we explain the abrupt injections observed in storms & McIlwain's data?
- The solution is topology. Abrupt changes in topology of the cusp also cause abrupt changes in output. The key is that the cusp is a POSITIVE feedback amplifier, and can be driven to the rails by appropriate input.
- The plasma trapped in the cusp also contributes to the magnetic field topology → making it stronger!

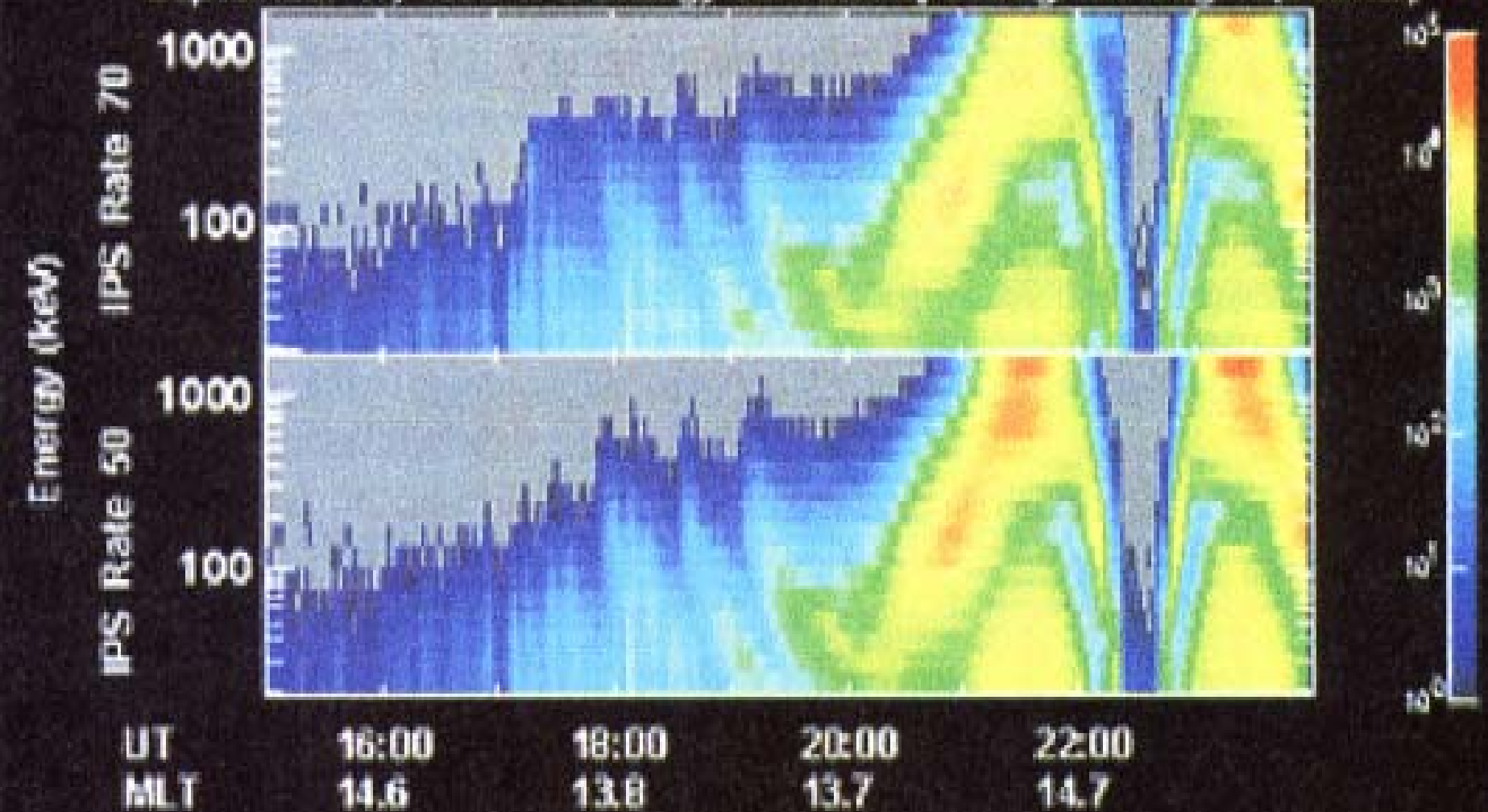


# Cusp Diamagnetic Cavities



# Energetic Energy Dispersed Events

September 17, 1996 IPS Energy vs. Time Spectrogram Log 10(counts)



# Model Predictions

- MeV electrons are born in CDCs
- Not solar wind  $V_{sw}$  but  $\Delta V_{sw}$  drives MeV e
- Conditions amenable to driving plasma in the cusp are the predictors for MeV e events:
  - Cusp tilted toward the Sun. Solstices over equinox.
  - High momentum solar wind.
  - $B_z$  northward during impact. Turbulence afterward.
- The longer the CDC lasts, the higher the “knee”, and the harder the spectrum
- MeV-Dst correlations are also due to  $\Delta V_{sw}$

# Scaling Laws

- $B_{\text{rad}} \sim B_{\text{surface}} = B_0$
- $B_{\text{cusp}} \sim B_0/R_{\text{stag}}^3$
- $E_{\text{rad}} = 5 \text{ MeV}$  for Earth
- $E_{\text{cusp}} \sim v_{\text{perp}}^2 \sim (B_{\text{cusp}}\rho)^2 \sim [(B_0/R_{\text{stag}}^3)R_{\text{stag}}]$
- $\mu = E/B$  is constant

$$E_{\text{rad-planet}} \sim (R_{\text{stag-Earth}}/R_{\text{stag-planet}})(B_{0\text{-planet}}/B_{0\text{-Earth}})^2 E_{\text{rad-Earth}}$$

# Scaled Radiation Belts

<u>Planet</u>	<u>R<sub>STAG</sub></u>	<u>B<sub>0</sub> (nT)</u>	<u>E<sub>RAD</sub></u>
Mercury	1.4	330	4 keV
Earth	10.4	31,000	5 MeV
Mars	1.25	< 6	< 1.5 eV
Jupiter	65	430,000	150 MeV
Saturn	20	21,000	1.2 MeV
Uranus	20	23,000	1.4 MeV
Neptune	25	14,000	0.42 MeV

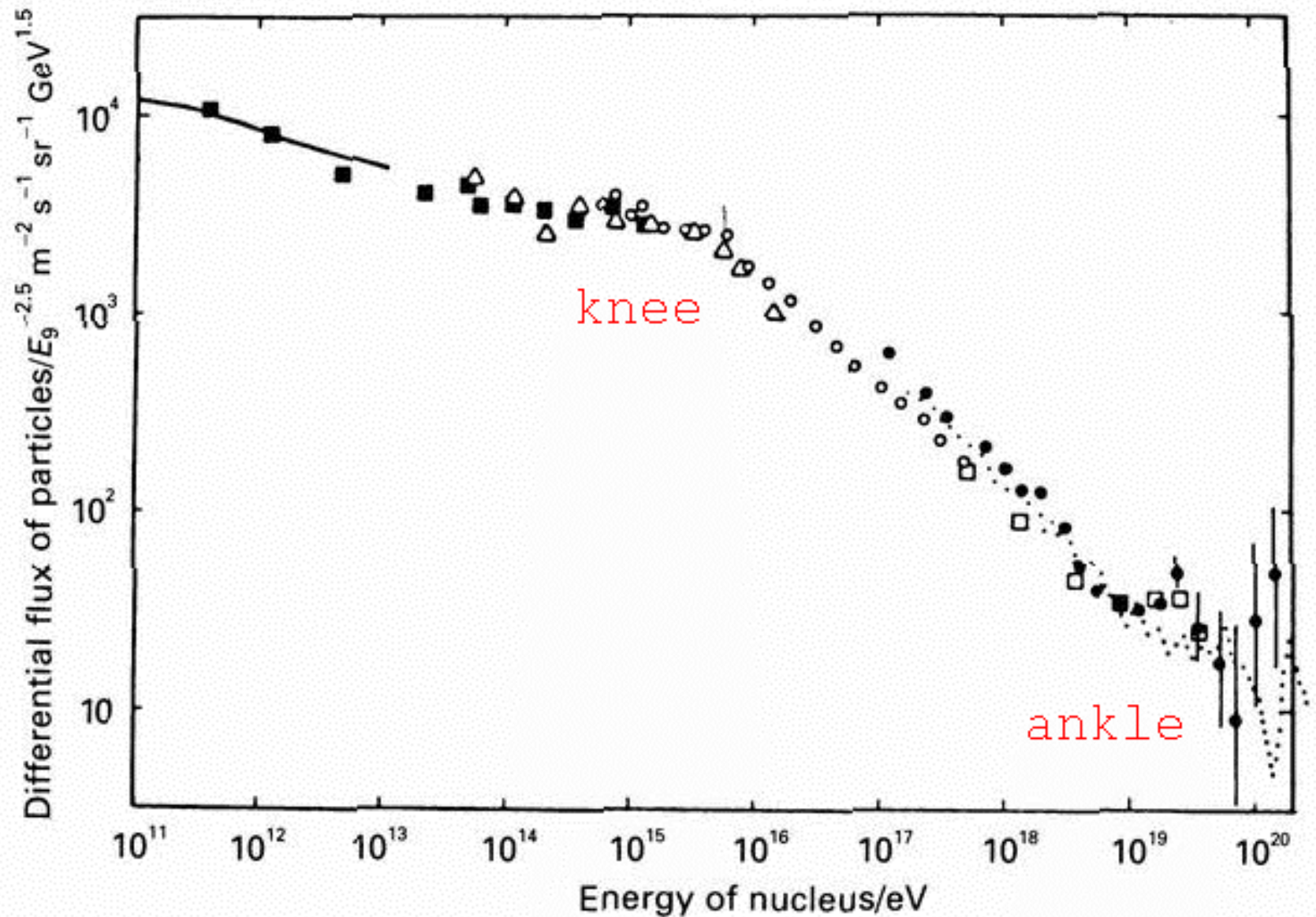


# More Predictions

- The unpredictability arises from a strong non-linearity in the trapping ability of the cusp. Small misalignment of cusp & no CDC is formed.
- Note that high  $V_{sw}$  usually means high  $\Delta V_{sw}$ , and simultaneously, higher seed energies for 2nd order Fermi acceleration. Thus a synergistic correlation.
- Magnetic clouds are also geoeffective, but with very different properties than high speed streams (Jan 97)
- CDC probably “evaporate” releasing lowest energy last, causing Time-Energy Dispersions.
- Positive feedback means many inputs=same output

## Problem #2: Source of GCR

# GCR Spectra



# Properties of GCR

- Energy density of GCR = 1 eV/cc ( $\sim 6$  @ galact. ctr)
- Energy density of Interstellar Medium components:
- GCR have equivalent energy to all other ISM stuff.

Phase	$f$	$n$ ( $\text{cm}^{-3}$ )	$kT$ (eV)	$nkT$ (eV $\text{cm}^{-3}$ )
Molecular Clouds	$10^{-3}$	$> 100$	$< 10^{-2}$	-
Cold Neutral Medium	0.025	40	$\approx 10^{-2}$	$\approx 0.4$
Warm Neutral Medium	$\approx 0.5$	$\approx 0.5$	$\approx 1$	$\approx 0.5$
Warm Ionized Medium	$\approx 0.25$	$\approx 0.2$	$\approx 1$	$\approx 0.2$
Hot Ionized Medium	$\approx 0.2$	$\approx 3 \times 10^{-3}$	$\approx 10^2$	$\approx 0.3$
Cosmic Rays	$\approx 1$	$\approx 10^{-9}$	$\approx 10^9$	$\approx 1$

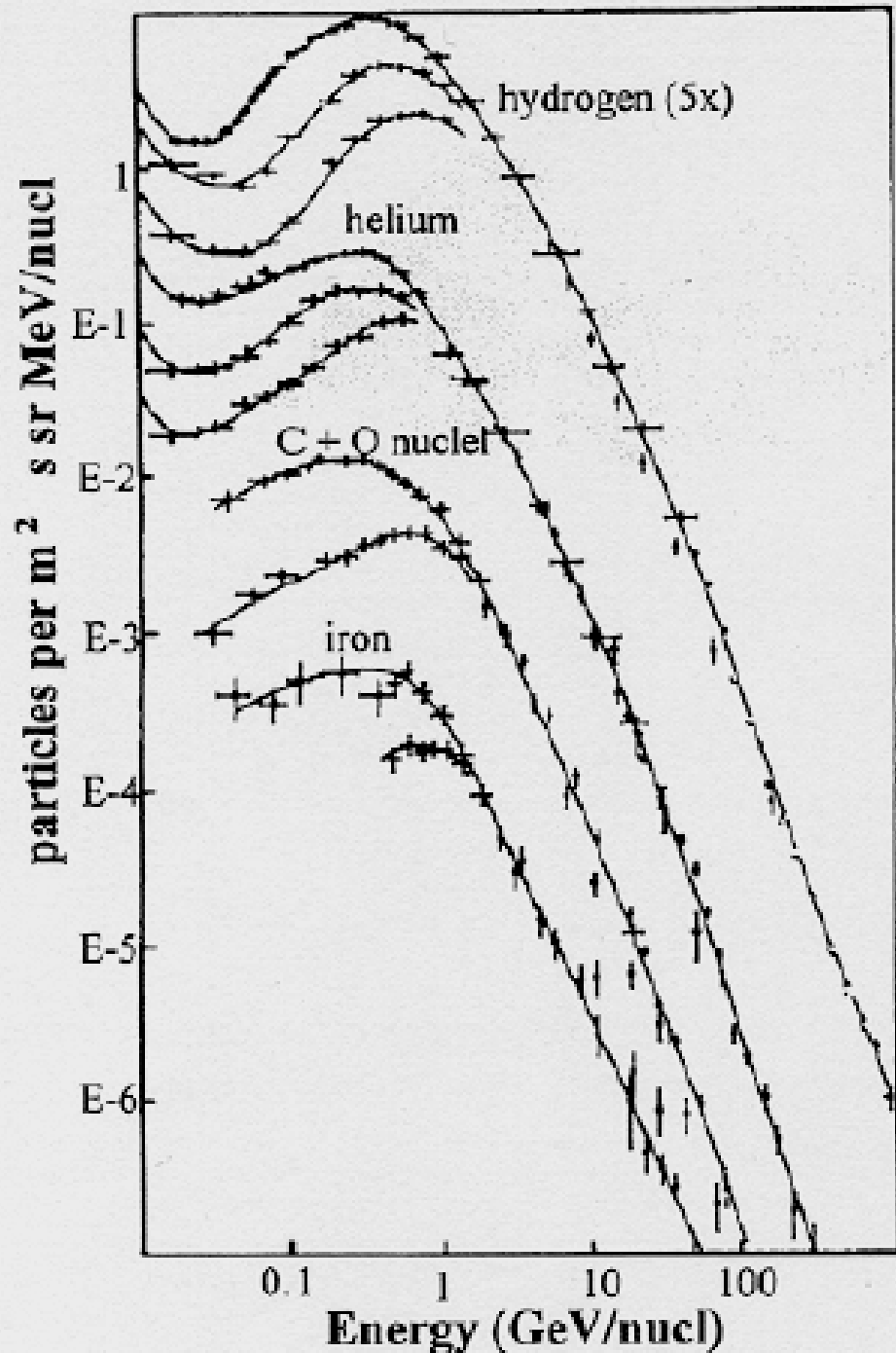
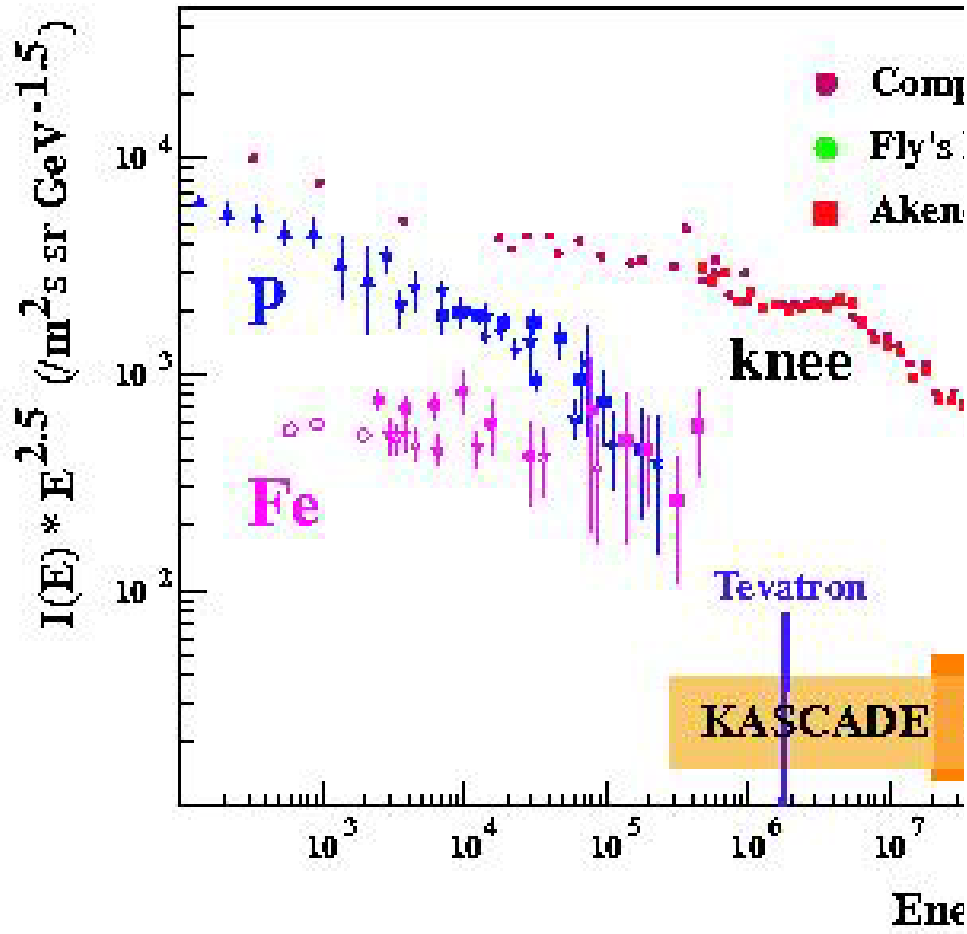
# Are GCR from Supernovae?

- Power output of Supernovae shock  $\sim 10^{51}/30\text{yr} = 10^{35}\text{W}$ , of which and estimated 15% show up in GCR, or  $2 \times 10^{34}$  Watts.
- Lifetime of GCR  $\sim 10^{15}\text{s}$ . (from  $^{10}\text{B}$  spallation)
- Energy Density \* Volume /Time =  $1\text{eV/cc} * 10^{69}\text{cc}/10^{15}\text{s}$   
 $= 3 \times 10^{35}\text{Watts}$  (and it only gets worse if you use the numbers in the galactic center) giving a ratio: SN/GCR = 0.1 !

Another calculation: Energy Density \* velocity \* area =  
luminosity  $\rightarrow 1\text{eV/cc} * 3 \times 10^{10}\text{cm/s} * 5 \times 10^{45}\text{cm}^2 = 1 \times 10^{35}\text{ Watts}$

- Even if the entire energy of a supernova went into GCR, and as we argued earlier, acceleration is a very inefficient process, we would still have an energy budget problem!
- (As some wag put it, SN are already highly oversubscribed, everyone already invokes it for their energy source)
- Where is the energy for GCR coming from?

# Low energy nuclei composition



# Some more peculiar coincidences

- Energy density of starlight  $= 0.3 \text{ eV/cc}$
- Energy density of ISM  $= \sim 1 \text{ eV/cc}$
- Energy density of interstellar B-fields  $= 0.2 \text{ eV/cc}$
- Cosmic Background Radiation  $= 0.3 \text{ eV/cc}$
- Nuclei 98%, electrons 2%
- Everyone calls these “coincidences”, but perhaps there is a theory that links them all together. My contention is that quadrupole cusp acceleration is just such a proto-theory.

# Quadrupole Cusps Pressure Balance

- Dimensional analysis: Energy/Vol  $\rightarrow$  Force/Area = Pressure. Thus mechanisms that equalize pressure will also equalize energy density.
- In a galaxy with a dipole magnetic field embedded in a flowing plasma, the cusp topology (and strength of the magnetic field) is affected by the ram pressure. Thus we can write an equilibrium:

$$P_{\text{GCR}} + P_{\text{mag}} + P_{\text{starlight}} = P_{\text{ram\_H}} + P_{\text{CMB}}$$

- Assuming that equipartition has balanced the IGM

$$P_{\text{ram\_H}} = P_{\text{CMB}}$$

- Thus we explain all these “coincidences” as a pressure equilibrium in the quadrupole cusp



# Energy Sources for GCR

- Where does the energy come from? Supernovae of course! Seriously, shock waves travelling out of the galactic disk transmit energy to the cusp & compress it, just as much as turbulence in the intergalactic medium (IGM). The cusp is a low-Q object, energy (waves) goes in, and doesn't come out.
- The advantage over Fermi-acceleration at SN? Continuous acceleration, multiple energy sources, identifiable rigidity properties. And the clincher...
- A natural explanation of the “knee”. At low-E, protons have the smaller rigidity, at high E, (due to gamma) Fe has the smaller rigidity. So they cross.

# Future Work

- Clearly this proto-theory is crying out for some modelling.
- Anomalous cosmic rays, coming from the heliopause shock, are also thought to be “shock accelerated”. If the GCR theory is correct, it should also apply to ACR. There is a great deal more data on ACR, which should make it a critical test case for the validity of the model.
- The Earth’s cusp is far more dynamic than either the heliosphere or galactosphere. Data from Earth satellites is most extensive. Work is needed to make a dynamic & predictive model of ORBE.

# Conclusion

- The Hamiltonian Dynamics of Quadrupole Traps show interesting features that permit rapid stochastic diffusion.
- Quadrupole traps have the right properties to make highly efficient accelerators at many length scales.
- The signatures of such accelerators--rigidity cutoffs, pressure balance, positive feedback--may resolve many outstanding problems in space & astrophysics.

*Soli Deo Gloria*